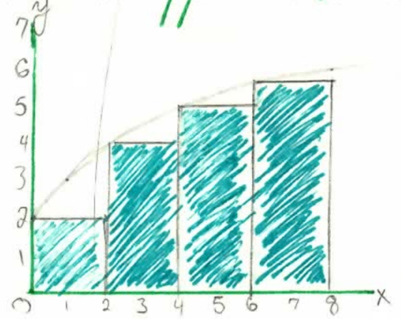
Skip Lester Homework 1/3/13
Calculus II Chapter 5.1

1) By Reading values of the given graph of f, use four rectangles to find a lower estimate and an upper estimate for the area under f from $x=0$ to $x=8$.



Lower Estimate:

$n=4$

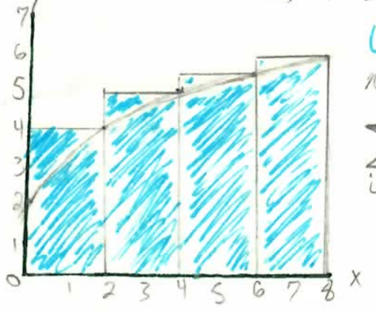
$$\sum_{i=1}^{4} f(x_i-1)\left(\frac{8-0}{4}\right)$$

$$= f(x_i-1)(2)$$

$$=(2\cdot2)+(4\cdot2)+(5\cdot2)+(5.5\cdot2)$$

$$4+8+10+11$$

$$12+21=\underline{33}$$



Upper-Estimate:

$n=4$

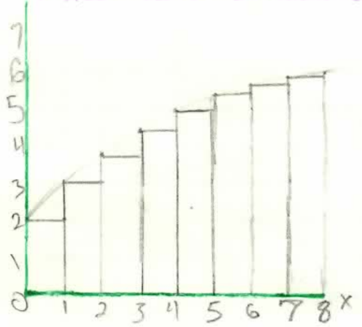
$$\sum_{i=1}^{4} f(x_i)(2)$$

$$=(4\cdot2)+(5\cdot2)+(5.5\cdot2)+(6\cdot2)$$

$$8+10+11+12=\underline{41}$$

$33 < area < 41$

Find new estimates using 8 rectangles.



Lower Estimate:

$n=8$

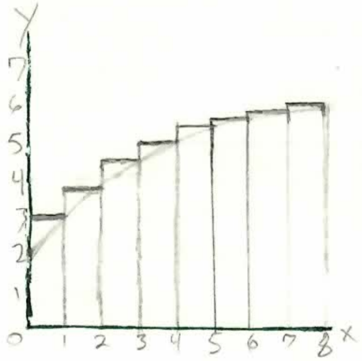
$$\sum_{i=1}^{8} f(x_i-1)\left(\frac{8-0}{8}\right)$$

$$=\sum_{i=1}^{8} f(x_i-1)$$

$$\approx 2+3+3.8+4.4+5+5.4+5.7+6=\underline{35.3}$$

$f(0)+f(1)+f(2)\ldots$

$35.3 < area < 39.2$ units^2



Upper Estimate:

$n=8$

$$\sum_{i=1}^{8} f(x_i)= 3+3.8+4.4+5+5.4+5.7+5.9+6=\underline{39.2}$$

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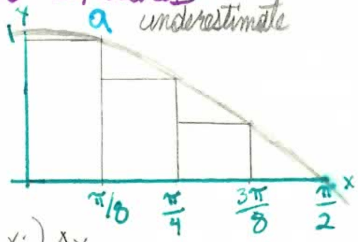
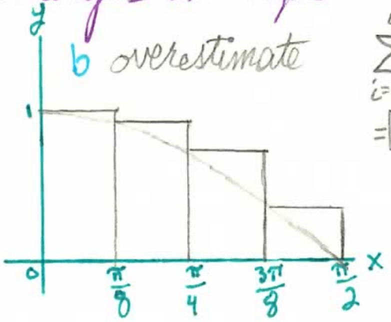
CALCULUS II Chapter 5.1

3) Estimate the area under the graph of $f(x) = \cos x$ from $x = 0$ to $x = \pi/2$ using four approximating rectangles and right end points and left on two sketches.

$\Delta x = \frac{\pi}{8}$





$$\sum_{i=1}^{4} f(x_i - 1)\Delta x$$
$$= \left[\cos 0 + \cos \tfrac{\pi}{4} + \cos \tfrac{3\pi}{8} + \cos \tfrac{\pi}{2}\right]\left(\tfrac{\pi}{8}\right)$$
$$\simeq \underline{1.1835} \text{ units}^2$$

$$= \sum_{i=1}^{4} f(x_i)\Delta x$$
$$= \left[\cos \pi/8 + \cos \pi/4 + \cos 3\pi/8 + \cos \pi/2\right](\pi/8) = \underline{.7908 \text{ units}^2}$$

⑦ For $N = 1000$, the program returned $.2005003\overline{3}$. I guess .2 is the exact area

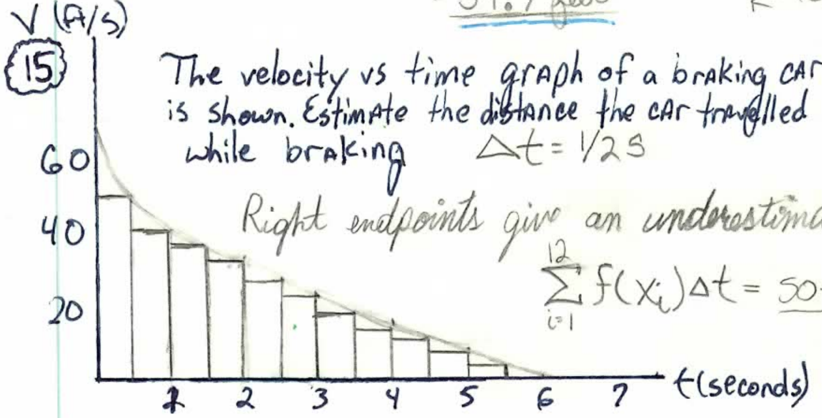
⑪ The speed of a runner increased steadily over the first three seconds of a race. Her speed is given in half-second intervals. Find upper & lower estimates for the distance she travelled.

$\Delta X = .5$ V is increasing, so left endpoints give us a lower estimate.

$$L = (0 + 6.2 + 10.8 + 14.9 + 18.1 + 19.4)(.5)$$
$$= 69.4(.5)$$
$$= \underline{34.7 \text{ feet}} \text{ lower}$$

$$R = (6.2 + 10.8 + 14.9 + 18.1 + 19.4 + 20.2)(.5)$$
$$= 89.6(.5)$$
$$= \underline{44.8 \text{ feet}} \text{ upper}$$

⑮ The velocity vs time graph of a braking car is shown. Estimate the distance the car travelled while braking $\Delta t = 1/2$ s



Right endpoints give an underestimate

$$\sum_{i=1}^{12} f(x_i)\Delta t = \frac{50+40+37+32+26+22+18+14+11+7+3+0}{2}$$
$$= \underline{130 \text{ feet}}$$

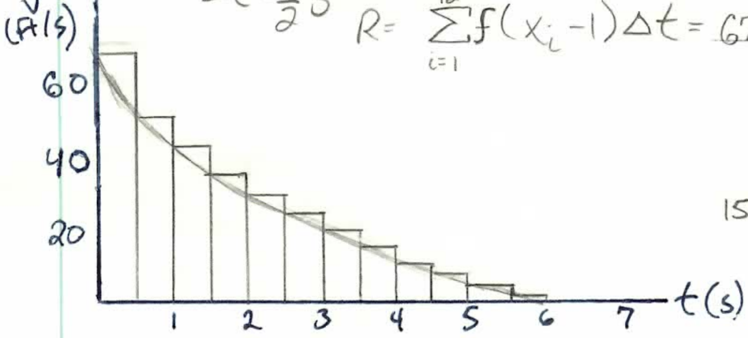
Left give an over-estimate

$\Delta t = \frac{1}{2}$ s

$$R = \sum_{i=1}^{12} f(x_i - 1)\Delta t = \frac{67+50+42+36+30+25+20+15+10+8+5+2}{2}$$
$$= 155 \text{ feet}$$

$155 + 130 = 285$

$\underline{285/2 = 142.5 \text{ feet}}$



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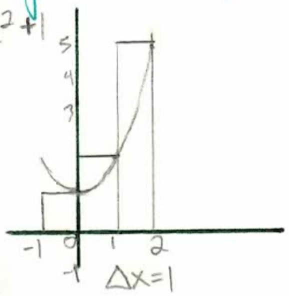
1/5/2013 Math 152 Chapter 5.1

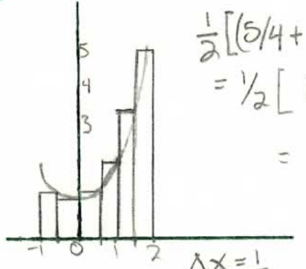
⑤ a) Estimate the area under the graph of $f(x) = 1 + x^2$ from $x=-1$ to $x=2$ using three rectangles and right endpoints. Then improve your estimate by using six.

$f(x) = x^2 + 1$

x	f(x)
-1	2
0	1
1	2
2	5



$\Delta x = 1$



$\Delta x = \frac{1}{2}$

$\frac{1}{2}[(5/4 + 1 + 5/4 + 2 + 13/4 + 5)]$

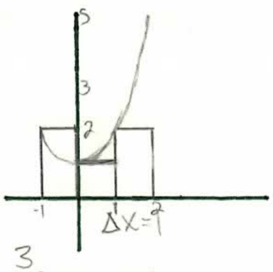
$= 1/2 [14/4 + 8/4 + 13/4 + 20/4]$

$= \frac{22 + 33}{8}$

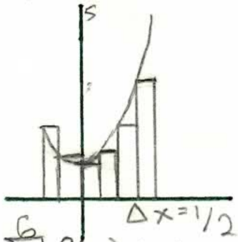
$= 55/8$

$\sum_{i=1}^{3} f(x_i)\Delta x = 1 + 2 + 5 = \underline{8}$ $\sum_{i=1}^{6} f(x_i)\Delta x = \underline{6.875}$

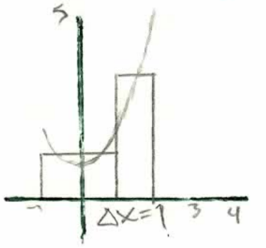
Next, use left endpoints and midpoints



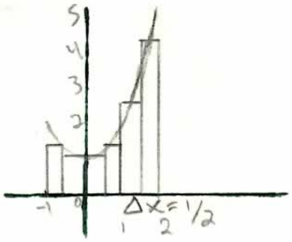
$\Delta x = 1$



$\Delta x = 1/2$



$\Delta x = 1$



$\Delta x = 1/2$

$\sum_{i=1}^{3} f(x_i - 1)\Delta x$

$2 + 1 + 2 = \underline{5}$

$\sum_{i=1}^{6} f(x)\Delta x$

$= 1/2[2 + 5/4 + 1 + 5/4 + 2 + 13/4]$

$= \frac{1}{2}\left[\frac{20}{4} + \frac{10}{4} + \frac{13}{4}\right]$

$= \underline{43/8}$

$\underline{5.375}$

$f(-.5) + f(.5) + f(1.5)$

$5/4 + 5/4 + 13/4$

$= \underline{23/4} = \underline{5.75}$

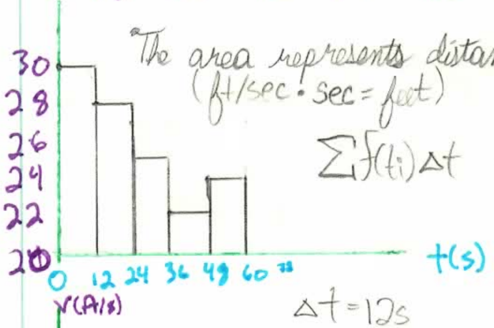
$1/2[f(-.75) + f(-.25) + f(.25) + f(.75) + f(1.25) + f(1.75)]$

$\frac{1}{2}[1.5625 + 1.0625 + 1.0625 + 1.5625 + 2.5625 + 4.0625]$

$= \underline{5.9375}$ ← Best estimate

⑫ Speedometer readings for a motorcycle at 12-second intervals are given.

t(s)	0	12	24	36	48	60
v(ft/s)	30	28	25	22	24	27

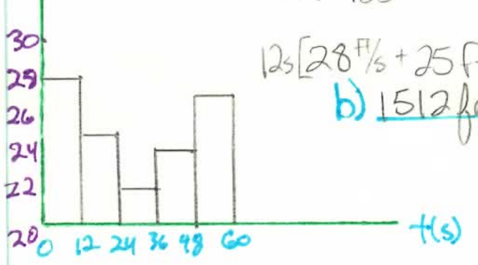


The area represents distance in feet (ft/sec · sec = feet)

$\sum f(t_i)\Delta t = (30 \cdot 12) + (28 \cdot 12) + (25 \cdot 12) + (22 \cdot 12) + (24 \cdot 12)$

a) 1548 feet

$\Delta t = 12s$



$12s[28 ft/s + 25 ft/s + 22 ft/s + 24 ft/s + 27 ft/s]$

b) 1512 feet

c) a is an overestimate, b is an underestimate.

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Math 152 Chptr 5.1

⑲ Use Definition 2 to find an expression for the area under the graph of f as a limit. Definition 2: $A = \lim_{n\to\infty} R_n = \lim_{n\to\infty}[f(x_1)\Delta x + \ldots + f(x_n)\Delta x]$

$f(x) = x\cos x \quad 0 \leq x \leq \pi/2$ $\quad A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

$\Delta x = \frac{(\pi/2 - 0)}{n} = \frac{\pi/2}{n}$

$x_i = 0 + i\Delta x \quad \frac{\pi}{2}\cdot\frac{i}{n}$

$x_i = \frac{\pi/2\, i}{n} = \frac{i\pi}{2n}$

$\underline{\lim_{n\to\infty} \sum_{i=1}^{n} (i\pi/2n)\cos(i\pi/2n)(\pi/2n)}$

㉑ Determine a region whose area is equal to the given limit:

$$\lim_{n\to\infty} \sum_{i=1}^{n} (\pi/4n)\tan(i\pi/4n)$$

tan x on $[0, \pi/4]$

$\Delta x = \frac{\pi/4}{n}$

$x_i = i\Delta x = \frac{i\pi}{4n}$ ∴ filling in $\lim_{n\to\infty} \sum_{i=1}^{n} f(x)\Delta x$, we have:

$\underline{\lim_{n\to\infty} \sum_{i=1}^{n} (\pi/4n)\tan(i\pi/4n)(\pi/4n)}$

㉓ Let A be the area under the graph of an increasing continuous function f from a to b, and let L_n and R_n be the approximations of A with n intervals and left and right endpoints, respectively

a) How are A, L_n, and R_n related?

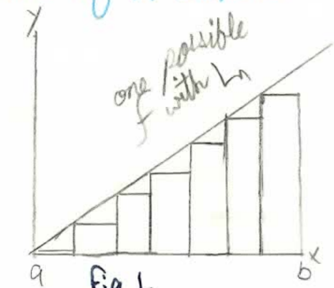


Fig 1.

Since we assume f is increasing on [a, b], L_n will always be an underestimate, and R_n will be an overestimate of the area A under f.

∴ $\underline{L_n < A < R_n}$ b) Show that $R_n - L_n = \frac{b-a}{n}[f(b) - f(a)]$

$R_n = f(x_1)\Delta x + f(x_2)\Delta x \ldots f(x_n)\Delta x$ $\quad R_n - L_n = f(x_n)\Delta x - f(x_0)\Delta x$

$L_n = f(x_0)\Delta x + f(x_1)\Delta x \ldots f(x_{n-1})\Delta x$ $\quad = \Delta x[f(x_n) - f(x_0)]$

$\quad = \frac{b-a}{n}[f(b) - f(a)]$ □

$\Delta x = (b-a)/n$ c) Deduce that

$R_n - A < \frac{b-a}{n}[f(b) - f(a)]$ Deduced! $L_n < A \therefore R_n - L_n > R_n - A$

Skip Lester Chapter 5.2

Math 152: Calc II 1/6/2013

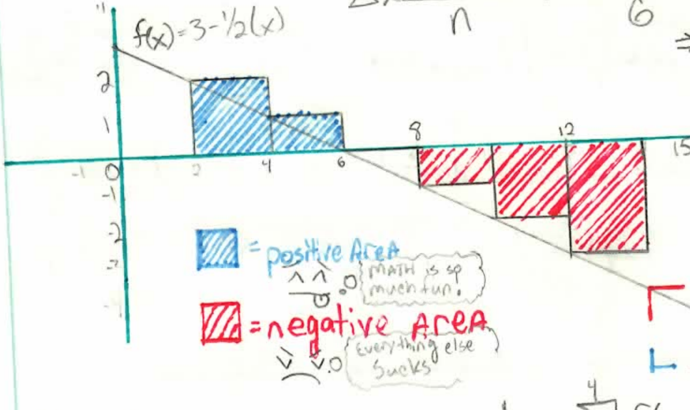
1, 5, 7, 9, 17, 19, 23

1. Evaluate the Reimann sum for $f(x) = 3 - \frac{1}{2}x$, $2 \le x \le 14$, with six subintervals, taking left end-points. Explain, with the aid of a diagram, what the Riemann sum represents.

5/5

$\Delta x = \frac{b-a}{n} \Rightarrow \frac{14-2}{6} = 2 \quad x_i = i-1$

$\Rightarrow \sum_{i=1}^{6} f(x_i - 1)\Delta x$



$= 2[(2)+(1)+(0)+(-1)+(-2)+(-3)]$
$= -6$

-6 units² represents the net area under $f(x) = 3 - \frac{1}{2}x$ from $x=2$ to $x=14$.

⑤ Attached printout

$x_i = 0 + \Delta x$

$\Delta x = (b-a)/n = (8-0)/4 = 2$

$R_4 = \sum_{i=1}^{4} f(x_i)\Delta x = 2[1+2+(-2)+1] = \underline{4}$

$L_4 = \sum_{i=1}^{4} f(x_{i-1})\Delta x$
$= 2[2+1+2+(-2)]$
$= \underline{6}$

$M_4 = \sum_{i=1}^{4} f(\bar{x})\Delta x$
$= 2[3+2+1+(-1)] = \underline{10}$

⑦ A table of values is given for a function F. Use the data to find lower and upper estimates for $\int_{10}^{30} f(x)\,dx$. f is increasing

$L_5 = \sum_{i=1}^{5} f(x_{i-1})\Delta x$

x	10	14	18	22	26	30
f(x)	-12	-6	-2	1	3	8

$\Delta x = \frac{30-10}{n} = \frac{20}{n} \quad n=5 \quad \Delta x = 4$

$R_5 \sum f(x)\Delta x$

$L_5 = 4[(-12)+(-6)+(-2)+1+3] =$
$\underline{4[-16] = -64}$

$R_5 = 4[(-6)+(-2)+1+3+8]$
$\underline{= 4[4] = 16}$

⑨ Use the midpoint rule with the given value of n to approximate the integral. Round the answer to four decimal places

$\int_2^{10} \sqrt{x^3+1}\,dx \quad n=4$

$\int_a^b f(x)dx \approx \sum_{i=1}^{n} f(\bar{x})\Delta x$

$\Delta x = \frac{10-2}{4} = 2$

where $\bar{x} = \frac{1}{2}(x_{i-1} + x_i)$

$= \frac{1}{2}(2+4) = 3$
$\frac{1}{2}(4+6) = 5$
$\frac{1}{2}(6+8) = 7$
$\frac{1}{2}(8+10) = 9$

f(x)	$\sqrt{28}$	$\sqrt{126}$	$\sqrt{344}$	$\sqrt{728}$
x	3	5	7	9

$2[(\sqrt{28} + \sqrt{126} + \sqrt{344} + \sqrt{728})] = \underline{124.1644}$

25, 27, 29, 31d 33-47odd

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Calculus II Chapter 5.2

(25) Use the form of the definite integral from theorem to evaluate:

$f(x) = \int_1^2 x^3 dx$ $\Delta x = \frac{2-1}{n} = \frac{1}{n}$ $x_i = 1 + (i/n)$

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x, \ \Delta x = \frac{b-a}{n} \ \& \ x_i = a + i\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} (1 + i/n)^3 (1/n)$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} (1 + i/n)^3$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(\frac{n+i}{n}\right)\left(\frac{n+i}{n}\right)\left(\frac{n+i}{n}\right)$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(\frac{n^2+2in+i^2}{n^2}\left(\frac{n+i}{n}\right)\right)$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left[\frac{n^3+2in^2+ni^2+in^2+2i^2n+i^3}{n^3}\right]$$

$$= \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left[\frac{n^3+3i^2n+3in^2+i^3}{n^3}\right]$$

$$= \lim_{n\to\infty} \frac{1}{n^4} \sum_{i=1}^{n} n^3+3in^2+3i^2n+i^3$$

$\sum_{i=1}^{n} c = nc$

$$= \lim_{n\to\infty} \frac{1}{n^4}\left[\sum_{i=1}^{n} \cdot n^3 + \sum_{i=1}^{n} 3n^2 i + \sum_{i=1}^{n} 3ni^2 + \sum_{i=1}^{n} i^3\right]$$

cramster

$$= \lim_{n\to\infty} \frac{1}{n^4}\left[n \cdot n^3 + 3n^2 \sum_{i=1}^{n} i + 3n \sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} i^3\right]$$

$$= \lim_{n\to\infty} \frac{1}{n^4}\left[n^4 + 3n^2\left(\frac{n(n+1)}{2}\right) + 3n\left(\frac{n(n+1)(2n+1)}{6}\right) + \left(\frac{n(n+1)}{2}\right)^2\right]$$

$$= \lim_{n\to\infty} \frac{1}{n^4}\left[n^4 + \frac{3n^4+3n^3}{2} + 3n\left(\frac{2n^3+3n^2+n}{6}\right) + \left(\frac{n^2+n}{2}\right)^2\right]$$

$$\frac{n^2+n}{2} \cdot \frac{n^2+n}{2} = $$
$$n^4+n^3+n^3+n^2$$
$$\frac{n^4+2n^3+n^2}{4}$$

$$= \lim_{n\to\infty} \frac{1}{n^4}\left[\frac{3n^4}{2} + \frac{6n^4}{6} + n^4 + \frac{3n^3}{2} + \frac{9n^3}{6} + \frac{3n^2}{6} + \frac{n^4}{4} + \frac{2n^3}{4} + \frac{n^2}{4}\right]$$

$$= \lim_{n\to\infty}\left[\frac{3}{2} + 1 + 1 + \frac{3}{2n} + \frac{3}{2n} + \frac{3}{6n^2} + \frac{1}{4} + \frac{2}{4n} + \frac{1}{4n^2}\right]$$

$$= \frac{6}{4} + \frac{8}{4} + \frac{1}{4} \qquad = \underline{\frac{15}{4}}$$

33-47 odds | Skip Lester Chapter 5.2
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~~(27)~~ (29) ~~(31)~~

27) Express the integral as a limit of Riemann's Sums

$\Delta x = \frac{6-2}{n} = \frac{4}{n}$ $x_i = a + i\Delta x$

$2 + \frac{4i}{n}$

$$\int_2^6 \frac{x}{1+x^5}\,dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{\left(2+\frac{4i}{n}\right)}{1+\left(2+\frac{4i}{n}\right)^5} \cdot \frac{4}{n}$$

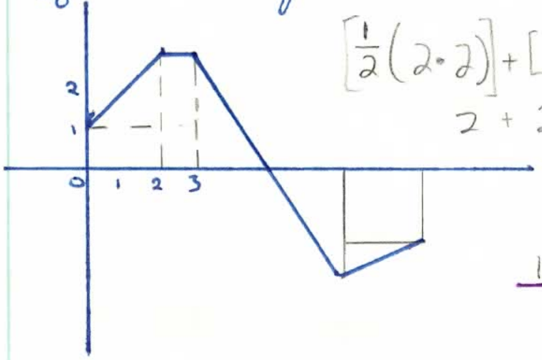
29) $\int_0^{\pi} \sin 5x\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \sin 5(x_i)\Delta x$

$\Delta x = \frac{\pi}{n}$ $= \lim_{n\to\infty} \sum \sin \frac{5i\pi}{n}\left(\frac{\pi}{n}\right)$

$i\Delta x = x_i$

$\frac{i\pi}{n} = x_i$

③① $\int_0^9 f(x)\,dx$ is given: Evaluate



$\left[\frac{1}{2}(2\cdot 2)\right] + [1\cdot 2] + [1\cdot 3] + \left[\frac{1}{2}(3\cdot 2)\right]$

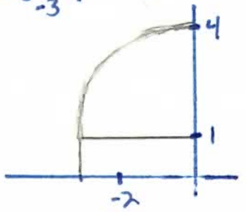
$2 + 2 + 3 + 3 = 10$ = positive area

$-\left[\left[\frac{1}{2}(2\cdot 3)\right] + (2\cdot 2) + \left[\frac{1}{2}(2\cdot 1)\right]\right]$

$3 + 4 + 1$

$10 - 8 = 2$

③⑤ $\int_{-3}^{0} \left(1 + \sqrt{9-x^2}\right) dx$



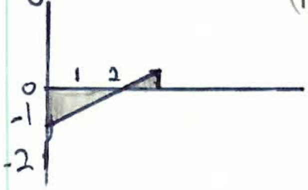
Quarter of Circle

$\frac{1}{4}\pi r^2 = \frac{1}{4}\pi 3^2$

$+[1\cdot 3]$

$\frac{1}{4}\pi 9 + 3 = \frac{9}{4}\pi + 3$

③③ $\int_0^3 \left(\frac{1}{2}x - 1\right) dx$



$(1/2)\left[\left(1\cdot\frac{1}{2}\right)\right] - (1/2)(2\cdot 1)$

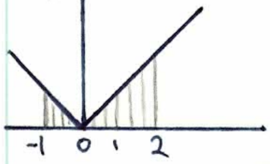
$\frac{1}{4} - 1 = -3/4$

(39) Evaluate $\int_{\pi}^{\pi} \sin^2 x \cos^4 x\,dx$

$\Delta x = \frac{\pi - \pi}{n} = 0 \therefore \int_{\pi}^{\pi} f(x)dx = 0$

③⑦ $\int_{-1}^{2} |x|\,dx = \left(\frac{1}{2}(-1\cdot 1)\right) + \left(\frac{1}{2}(2\cdot 2)\right)$

$1/2 + 2 = 5/2$



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Calculus II 1/8/2013

41) $\int_{-2}^{2} f(x)dx + \int_{2}^{5} f(x)dx - \int_{-2}^{-1} f(x)dx$

Write as a single integral in the form $\int_a^b f(x)dx$ pg. 351

$\int_{-2}^{5} f(x)dx + \int_{-1}^{-2} f(x)dx \rightarrow \int_{-1}^{5} f(x)dx$

property 5 + reverse limits

43) If $\int_0^9 f(x)dx = 37$ and $\int_0^9 g(x)dx = 16$, find $\int_0^9 [2f(x) + 3g(x)]\,dx$.

$(2 \cdot 37) + (3 \cdot 16) = \underline{122}$

45) evaluate $\int_1^3 e^{x+2}dx = \lim_{n \to \infty} \frac{2}{n} \sum_{i=1}^{n} e^{3+2i/n}$

$$\int_1^3 e^{x+2}dx = \int_1^3 (e^x \cdot e^2)\,dx$$
$$= e^2 \int_1^3 e^x dx$$
$$= e^2(e^3 - e)$$
$$= e^5 - e^3$$

47) $B < E < A < D < C$

$F(b) - F(a) \Rightarrow$ where $F' = f$

1-33 odds, 41, 49, 53

Skip Lester 1/9/2012

Calculus II Chapter 5.3

5/5

perfect

1) $\int_{-2}^{3}(x^2-3)\,dx \quad -f=(x^2-3)$

$\Rightarrow \qquad F' = \frac{1}{3}x^3 - 3x$

$= \left[\frac{1}{3}(3)^3 - 3(3)\right] - \left[\frac{1}{3}(-2)^3 - 3(-2)\right]$

$= \frac{1}{3}(27) - 9 \quad - \frac{1}{3}(-8) + 6$

$= 9 - 9 \quad + \left[\frac{8}{3} - 6\right]$

$= +\frac{8}{3} - \frac{18}{3} = \underline{\frac{-10}{3}}$

3) $\int_0^2 (x^4 - \frac{3}{4}x^2 + \frac{2}{3}x - 1)\,dx$

$\rightarrow \frac{1}{5}x^5 - \frac{1}{4}x^3 + \frac{1}{3}x^2 - x\Big]_0^2 = F$

$F(2) - F(0)$

$= \frac{1}{5}2^5 - \frac{1}{4}(2)^3 + \frac{1}{3}(2)^2 - 2 - 0 =$

$= \frac{32}{5} - 2 + \frac{4}{3} - 2 =$

$\frac{96}{15} + \frac{20}{15} - \frac{60}{15} = \underline{\frac{56}{15}} \quad 3\frac{11}{15}$

5) $\int_0^1 x^{4/5}\,dx$

$F = \frac{5}{9}x^{9/5}$

$F(1) - F(0) =$

$\frac{5}{9}(1) - 0 = \underline{5/9}$ ✓

7) $\int_{-1}^{0}(2x - e^x)\,dx$

$= \int_{-1}^{0} 2x\,dx - \int_{-1}^{0} e^x\,dx$

$= \quad x^2 - e^x = F$

$\underline{\int_{-1}^{0} f(x)\,dx = -2 + \frac{1}{e}}$

$F(b) - F(a) \Rightarrow 0^2 - e^0 - [-1^2 - e^{-1}]$

$F(0) - F(-1) \quad =$

$-1 - 1 + \frac{1}{e}$

$= -2 + (1/e)$

9) $\int_1^2 (1+2y)^2\,dy$

$F = (1+2y)(1+2y)$

$= 1 + 4y + 4y^2$

$F = \frac{4}{3}y^3 + 2y^2 + y$

$\frac{d}{dx}F(g(x))$

$\Rightarrow$

$F'(g(x))g'(x)$

$F(2) - F(1)$

$(4/3)(8) + 8 + 2 - [(\frac{4}{\ }) + (6/3) + (3/3)]$

$13/3$

$\frac{32}{3} + \frac{24}{\ } + \frac{6}{3} - \left[\frac{13}{3}\right]$

$\frac{62 - 13}{3} = \underline{\frac{49}{3}}$ ✓

1) $\int_1^9 \frac{x-1}{\sqrt{x}}\,dx$

$=$

$= \int_1^9 \frac{x}{\sqrt{x}}dx - \int_1^9 \frac{1}{\sqrt{x}}\,dx$

$= \int_1^9 \sqrt{x}\,dx - \int_1^9 \frac{1}{\sqrt{x}}\,dx$

$= \int_1^9 x^{1/2}dx - \int_1^9 x^{-1/2}dx$

$F = \frac{2}{3}x^{3/2} - 2x^{1/2}$

$= \frac{2}{3}\sqrt{x^3} - 2\sqrt{x}$

$F(9) - F(1)$

$\Rightarrow \frac{2}{3}\sqrt{9^3} - 2\sqrt{9} \quad - \left[\frac{2}{3}\sqrt{1^3} - 2\sqrt{1}\right]$

$\frac{2}{3}(27) - 6 - \left[\frac{2}{3} - 2\right]$

$18 - 6 + 4/3 \quad = \underline{40/3}$

$12 + 4/3$

13-33 odd, 41, 49, 53

Skip LesteR Math 152 Calc II
1/9/2013 Chapter 5.3

13) $\int_0^1 x(\sqrt[3]{x} + \sqrt[4]{x})\,dx$

$= x^{4/3} + x^{5/4}$

$\int_0^1 x^{4/3} + x^{5/4}$

$\Rightarrow \frac{3}{7}x^{(7/3)} + \frac{4}{9}x^{(9/4)} = F$

$\int_0^1 f(x)\,dx = F(b) - F(a)$
$= F(1) - F(0)$

$= \frac{3}{7} + \frac{4}{9} - 0 = \underline{\frac{55}{63}}$

15) $\int_0^{\pi/4} \sec^2 t\,dt$

$F = \tan t$

$\Rightarrow F(\pi/4) - F(0)$

$= \tan(\pi/4) - \tan(0)$

$= 1 - 0 = \underline{1}$

17) $\int_1^9 \frac{1}{2x}\,dx$

$= \frac{1}{2}\int \frac{1}{x}\,dx$

$\Rightarrow \frac{1}{2}\left[\ln x\right]_1^9$

$= 1/2 \ln 9 - 1/2 \ln 1$

$= \frac{1}{2}\ln - 0$

$= \ln 9^{1/2} = \ln\sqrt{9}$

$= \underline{\ln 3}$

⑲ $\int_{1/2}^{\sqrt{3}/2} \frac{6}{\sqrt{1-t^2}}\,dt$

$= 6\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-t^2}}\,dt$

$\Rightarrow 6\left[\sin t\right]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}$

$= 6\sin^{-1}(\sqrt{3}/2) - 6\sin^{-1}(1/2)$

$= 6\frac{\pi}{3} - 6\frac{\pi}{6}$

$= 2\pi - \pi = \underline{\pi}$

㉑ $\int_{-1}^1 e^{u+1}\,du$

$\Rightarrow \left[e^{u+1}\right]_{-1}^1$

$= e^{1+1} - e^{-1+1}$

$= e^2 - e^0$

$= \underline{e^2 - 1}$

㉓ $\int_1^2 \frac{v^3+3v^6}{v^4}\,dv$

$= \int_1^2 \frac{v^3}{v^4} + \frac{3v^6}{v^4}\,dv$

$= \int_1^2 \frac{1}{v} + 3v^2\,dv$

$= \left[\ln|v| + v^3\right]_1^2$

$= [\ln 2 + 2^3] - [\ln 1 + 1^3]$

$= \ln 2 + 8 - [0 + 1]$

$= \underline{\ln 2 + 7}$

㉕ $\int_0^{\frac{\pi}{4}} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta$

$= \int_0^{\pi/4} \left[\frac{1}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}\right] d\theta$

$= \int_0^{\pi/4} \sec^2\theta + 1\,d\theta$

$= \left[\tan\theta + \theta\right]_0^{\frac{\pi}{4}}$

$= \tan(\pi/4) + \pi/4 - [\tan 0 + 0]$

$= \underline{1 + \pi/4}$

$\boxed{27}$ $\int_0^{1/\sqrt{3}} \frac{t^2-1}{t^4-1}\,dt$

$= \int_0^{\frac{1}{\sqrt{3}}} \frac{\cancel{t^2-1}}{\cancel{(t^2-1)}(t^2+1)}$

$= \int_0^{\frac{1}{\sqrt{3}}} \frac{1}{t^2+1}$

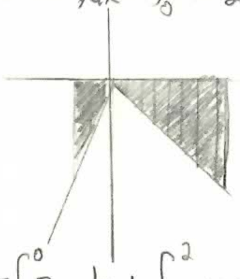
$= \left[\tan^{-1}(t)\right]_0^{\frac{1}{\sqrt{3}}}$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \tan^{-1}(0)$

$= \underline{\pi/6}$

29) $\int_{-1}^2 (x - 2|x|)\,dx$

$\int_{-1}^0 x - 2(-x)\,dx + \int_0^2 x - 2x\,dx$



$= \int_{-1}^0 3x\,dx + \int_0^2 -x\,dx$

$= \int_{-1}^0 3x\,dx - \int_0^2 x\,dx$

$= \left[\frac{3}{2}x^2\right]_{-1}^0 - \left[\frac{x^2}{2}\right]_0^2$

$F = 3\left[\frac{x^2}{2}\right]_{-1}^0 - \left[\frac{x^2}{2}\right]_0^2$

$= (0 - \frac{3}{2}) - (\frac{4}{2} - 0) = -\frac{7}{2}$

31, 33, 41, 49, 53

Skip Lester 1/9/2013
Calculus II Chapter 5.3

31) What is wrong with

$$\int_{-1}^{3}\frac{1}{x^2}\,dx = \frac{x^{-1}}{-1}\Big]_{-1}^{3} = \frac{-4}{3}$$

$\frac{1}{x^2}$ is not defined for $x=0$; The evaluation theorem cannot be used.

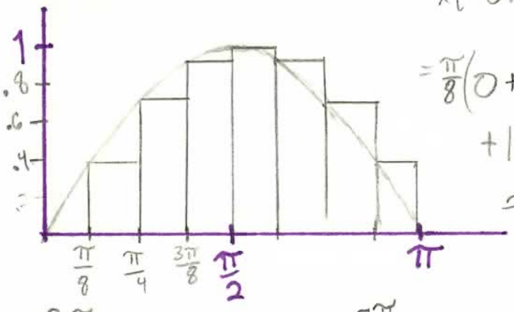
33) $y = \sin x,\ 0 \le x \le \pi$

$\Delta x = \frac{\pi - 0}{8} = \frac{\pi}{8}$

$x_i = 0 + \frac{\pi i}{8}$ $\quad \sum_{i=1}^{9} \sin\frac{(i-1)\pi}{8}\left(\frac{\pi}{8}\right)$

$= \frac{\pi}{8}(0 + .382 + .707 + .92 + 1 + .92 + 707 + .382)$

$\simeq \underline{1.9743}$



$$\int_0^{\pi} \sin x\,dx = -\cos x\Big]_0^{\pi}$$

$-\cos\pi - -\cos 0$

$1 + 1 = \underline{2}$

41) Find the general indefinite integral.

$$\int\left(\cos x + \frac{1}{2}x\right)dx$$

$\underline{\sin x + \frac{1}{4}x^2 + C}$

49) $\int_0^2 (2y - y^2)\,dy$

$\rightarrow y^2 - \frac{y^3}{3}\Big]_0^2$

$= 4 - (8/3) - 0 - 0$

$= \frac{12}{3} - \frac{8}{3} = \underline{\frac{4}{3}}$

53) If oil leaks from a tank at a rate of $r(t)$ gallons per minute at time t, what does $\int_0^{120} r(t)\,dt$ represent?

The integral of a rate gives us the net change. that is the gallons leaked over 120 minutes, or 2 hours from $t=0$ hours. Net Change Theorem

46, 47, 59-63 odds 67

Skip Lester 1/10/2011
Calculus II Chapter 5.3

46) $\int \sec t(\sec t + \tan t)\,dt$

$\int \sec^2 t + \sec t \tan t \, dt$

$\Rightarrow \underline{\tan t + \sec t + C}$

47) $\int \frac{\sin x}{1-\sin^2 x}\,dx$

$= \int \frac{\sin x}{\cos^2 x}\,dx$

$= \int \frac{\sin x}{\cos x \ \cos x}\,dx$

$= \int \tan x \sec x \, dx$

$\Rightarrow \underline{\sec x + C}$

⑸⑼ Find the displacement of a particle

a) moving along a line according to the velocity function

$v(t) = 3t - 5 \quad 0 \leq t \leq 3$

$\int_0^3 3t - 5 \rightarrow \frac{3}{2}t^2 - 5t \Big]_0^3$

$\frac{3}{2}(9) - (5 \cdot 3)$

$\frac{27}{2} - \frac{30}{2} = \underline{-\frac{3}{2}}\ m$

b) Find the distance travelled by the particle during the interval

Distance $= \int_{t_i}^{t_f} |v(t)|\,dt$

Property 5 of integrals

$v(t) \leq 0 \Leftrightarrow t \leq 5/3$

$v(t) > 0 \Leftrightarrow t > 5/3$

$\therefore -\int_0^{5/3} v(t)dt + \int_{5/3}^{3} v(t)dt = \text{distance}$

$= -\left[\frac{3}{2}\left(\frac{5}{3}\right)^2 - 5\left(\frac{5}{3}\right)\right] - 0 + \left[\left[\frac{3}{2}(3^2) - (5 \cdot 3)\right] - \left[-\frac{25}{6}\right]\right]$

$-1 \cdot \left(\frac{3}{2}\left(\frac{25}{9}\right) - \frac{25}{3}\right) - 0 + \left[\left[-\frac{3}{2}\right] + \frac{25}{6}\right]$

$-1 \cdot \left(\frac{75}{18} - \frac{150}{18}\right) - 0 \qquad \frac{-9}{6} + \frac{25}{6}$

$-\left[\frac{-25}{6}\right] + \frac{25}{6} - \frac{9}{6}$

$\frac{50}{6} - \frac{9}{6} = \underline{\frac{41}{6}}\ m$

61, 63, 67

Skip Lester 1/10/11
Calculus II Chapter 5.3

61) The acceleration function and initial velocities are given for a particle moving along a line. Find the velocity at time t and the distance travelled during the time interval. $a(t) = t+4$, $v(0)=5$ $0 \le t \le 10$

$\int_0^{10}(t+4)\,dt \rightarrow \frac{t^2}{2}+4t\Big]_0^{10}$ $\frac{0^2}{2}+4(0)+C=5$

$C=5$

$\therefore v(t) = \frac{t^2}{2}+4t+5$

$\frac{t^2}{2}-4t+5<0$
$-5 \quad -5$

$\frac{t^2}{2}-4t<-5$

v(t) is positive in our domain

$\int_0^{10} \frac{t^2}{2}+4t+5 \rightarrow \frac{t^3}{6}+2t^2+5t\Big]_0^{10}$

$\therefore r(t) = \frac{t^3}{6}+2t^2+5t$ $R(10)-R(0)$

$= r(10) - 0$

$= \frac{10^3}{6}+2(10^2)+5(10) = \frac{1000}{6}+200+50 = \frac{2500}{6}$

$= \frac{1000+1200+300}{6} = \frac{1250}{3}\,m$

63) The linear density of a rod of length 4m is given by $P(x) = 9+2\sqrt{x}$, measured in kg/m, where x is measured in meters from one end of the rod. Find the total mass of the rod.

$\int_0^4 9+2\sqrt{x} \Rightarrow 9x+\frac{4}{3}x^{3/2}\Big]_0^4$

$= \int_0^4 9+2(x^{1/2}) = \frac{4}{3}\sqrt{x^3}+9x\Big]_0^4$

$\rightarrow F(4)-F(0)$

$= \frac{4}{3}\sqrt{4^3}+36$

$= \frac{4}{3}\sqrt{64}+36$

$= \frac{4}{3}(8)+36$

$= \frac{32}{3}+36 = 46\frac{2}{3}\,Kg$

5.3 #67

Skip Lester 1/10/13
CALCULUS II

67) The marginal cost of manufacturing x yards of a certain fabric is

$C'(x) = 3 - .01x + .000006x^2$ (in \$/yard)

$3 - \frac{x}{100} + \frac{6x^2}{1000000}$

Find the increase in cost if the production level is raised from 2000 yards to 4000 yards. $C(4000) - C(2000) = \int_{2000}^{4000} C'(x)\,dx$

$$3x - \frac{x^2}{200} + \frac{2x^3}{100\,0000}\Bigg]_{2000}^{4000} = 12{,}000 - \frac{(4{,}000)^2}{200} + \frac{2(4{,}000)^3}{1{,}000{,}000} = \$60{,}000$$

$$- \; 6{,}000 - \frac{2{,}000^2}{200} + \frac{2(2{,}000^3)}{1{,}000{,}000} = \$\,2000$$

$$\$\underline{58{,}000.^{00}}$$

Chapter 5.4
#3-19 odd

Skip Lester
Calculus II 1/10/13

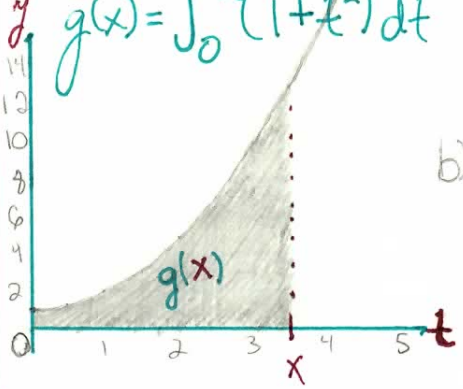
③ Let $g(x)=\int_0^x f(t)\,dt$

a) Evaluate $g(0)$, $g(1)$, $g(2)$, $g(3)$, and $g(6)$.

$\underline{g(0)=0}$ $\underline{g(1)=2}$, $\underline{g(2)=5}$, $g(3)=5+\frac{1}{2}(1)(4)$ $\underline{=7}$ $g(6)=7-4=\underline{3}$

⑤ Sketch the area represented by $g(x)$. Find $g'(x)$ using a) FTC1 b) evaluate the integral and then differentiate.

$g(x)=\int_0^x (1+t^2)\,dt$



$\boxed{1}$ FOR $g(x)=\int_a^x f(t)\,dt$ where f is continuous on $[a,b]$

$g=F$, $g'=f$ $\therefore$ a) $g'(x)=\underline{f(t)=(1+t^2)}$

b) $\int_0^x (1+t^2) \rightarrow t+\frac{t^3}{3}\Big]_0^x = \overset{F(B)-F(A)}{[x+(x^3/3)]-0}$

$\frac{d}{dx}\ x+\frac{x^3}{3} \Rightarrow 1+3x^2/3=x^2+1$ □

⑦ Use part 1 of the fundamental theorem of Calculus to find the derivative of the function: $g(x)=\int_1^x \frac{1}{t^3+1}\,dt$ $\boxed{1}$ $g'=f$, here $f=\underline{\frac{1}{t^3+1}}$ $\therefore g'=\underline{\frac{1}{x^3+1}}$, $x\in(-1,\infty)$

⑨ $g(y)=\int_2^y t^2 \sin t\,dt \rightarrow g'(y)=f(t)=\underline{y^2\sin y}$

⑪ $F(x)=\int_x^{\pi}\sqrt{1+\sec t}\,dt$ $F'=f \Rightarrow -\frac{d}{dx}\int_{\pi}^{x}\sqrt{1+\sec t}\,dt=\underline{-\sqrt{1+\sec t}}$

$=-\int_{\pi}^{x}\sqrt{1+\sec t}\,dt$

⑬ $h(x)=\int_2^{1/x} \arctan(t)\,dt$ let $u=1/x$ $\frac{du}{dx}=-1/x^2$ $\frac{dh}{dx}=\frac{dh}{du}\frac{du}{dx}$

$h'(x)=\frac{d}{du}\int_2^u \arctan(t)\,dt\,\frac{du}{dx}$

$=\arctan u\,\frac{du}{dx}$

$=\tan^{-1}u\cdot\frac{-1}{x^2}$

$=\underline{-\frac{\tan^{-1}(1/x)}{x^2}}$

Find the derivative of the given function

Chapter 5.4
15, 17, 19

Skip Lester Calculus II
January 13, 2013

⑮ $y=\int_0^{\tan x}\sqrt{t+\sqrt{t}}\,dt$ Let $u=\tan x$ $\frac{du}{dx}=\sec^2 x$

$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

$y=\int_0^u\sqrt{t+\sqrt{t}}\,dt\cdot\frac{du}{dx}$

$\Rightarrow y'=\sqrt{u+\sqrt{u}}\,\frac{du}{dx}$

$=\underline{\sqrt{\tan x+\sqrt{\tan x}}\cdot\sec^2 x}$

⑰ $g(x)=\int_{2x}^{3x}\frac{u^2-1}{u^2+1}\,du$ Let v= do this

$=\int_{2x}^{0}\frac{u^2-1}{u^2+1}du+\int_0^{3x}\frac{u^2-1}{u^2+1}du$

$=-\int_0^{2x}\frac{u^2-1}{u^2+1}du+\int_0^{3x}\frac{u^2-1}{u^2+1}du$

$g'\Rightarrow-\left(\frac{(2x)^2-1}{(2x)^2+1}\cdot\left[\frac{d}{dx}2x\right]\right)+\left(\frac{(3x)^2-1}{(3x)+1}\cdot\left[\frac{d}{dx}3x\right]\right)$

$=-2\left(\frac{4x^2-1}{4x^2+1}\right)+3\left(\frac{9x^2-1}{9x^2+1}\right)$

$=-\frac{8x^2-2}{8x^2+2}+\frac{27x^2-3}{27x^2+3}$

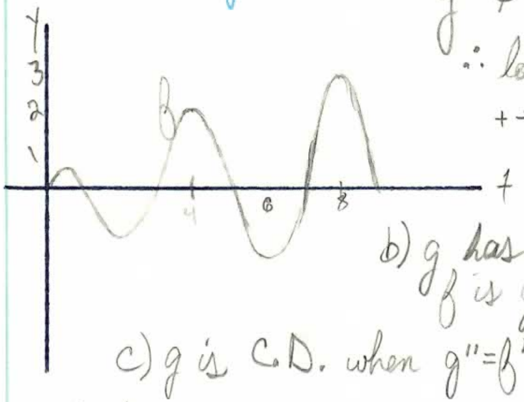
⑲ Let $g(x)=\int_0^x f(t)dt$, where f is the function of the given graph.

a) at what values do the local ~~vo~~ maximum & minimum values of g occur?

b) where does g gain its absolute maximum?

c) on which intervals is g concave downward?

d) Sketch g



a) The maxima and minimum values of g occur when $g'=0$, or, since by FTC1, $g'=f$, when $f=0$

∴ local maxima & minima occur at $x=1,3,5,7,\,\&\,9$

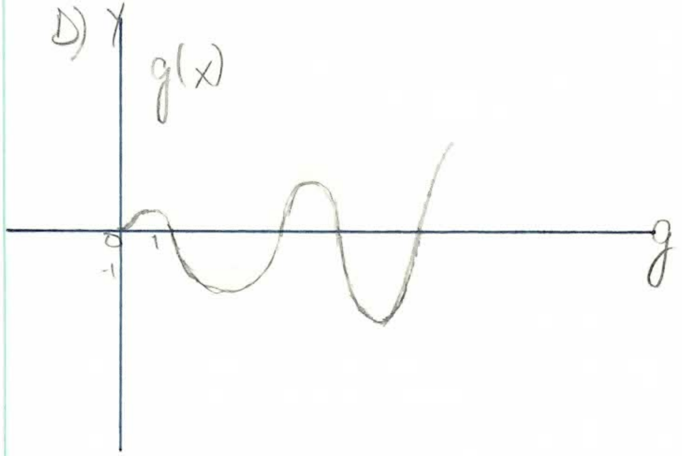
$+\rightarrow-\Leftrightarrow x=1\,\&\,5$, so g has maxima at $g(1)\,\&\,g(5)$

$-\rightarrow+\Leftrightarrow x=3\,\&\,7$, ∴ g has minima at $g(3), g(7)$

b) g has local maximum at $x=9$, when the area under f is greatest.

c) g is C.D. when $g''=f'<0$. From the graph of f, $f'<0$ on $(\frac{1}{2},2)\cup(4,6)\cup(8,9)$

D)



Skip Lester Calculus II

January 14, 2013 21, 23, 24, 27

(21) If $f(x) = \int_0^x (1-t^2) e^{t^2} dt$, on what interval is f increasing?

f is increasing when $f' > 0$; $\frac{d}{dx}$ of an integral of a function is the function.

$\frac{d}{dx} \int_a^x f(t)dt = f(x)$ ∴ f is increasing ⇔ $(1-x^2)e^{x^2} > 0$ $1-x^2 > 0$ $e^{x^2} > 0$ $x \in \mathbb{R}$

$-x^2 > -1$

$x^2 < 1$

$x < \pm\sqrt{1} = \pm 1$; $x \in (-1,1)$

∴ f is increasing on (-1,1)

(23) On what interval is the curve $y = \int_0^x \frac{t^2}{t^2+t+2} dt$ concave downward?

y is Concave downward when $y'' < 0$; y'' is given by $\frac{d}{dt} \frac{t^2}{t^2+t+2}$ $\Rightarrow$ let $f(x) = t^2, g(x) = t^2+t+2$

$$\frac{(t^2+t+2)(2t) - (t^2)(2t+1)}{(t^2+t+2)^2}$$

$x(x+4) < 0$ ⇔

∴ y'' is negative on $x \in (-4,0)$

$$= \frac{\cancel{2t^3} + 2t^2 + 4t - \cancel{2t^3} - t^2}{(t^2+t+2)^2}$$

$$= \frac{t^2+4t}{(t^2+t+2)^2} = \frac{x(x+4)}{(x^2+x+2)^2} = y''$$

	x	(x+4)	f(x)
-5	-	-	+
-2	-	+	-
2	+	+	+

(24) Find the slope of the tangent line to the curve with parametric equations $x = \int_0^t \sqrt{1+u^3}\,du$, $y = 1 + 2t + (-t)^3$ at point (0,1)

$y' = -3t^2 + 2$

$x' = \sqrt{1+t^3}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$0 = \int_0^t \sqrt{1+u^3} \rightarrow t = 0$

$1 = 1 + 2t - t^3$

$-1 \quad -1$

$0 = -t^3 + 2t$

$0 = t(-t^2+2)$

$$\frac{dy}{dx} = \frac{-3t^2+2}{\sqrt{1+t^3}} \Rightarrow$$

$$m = \left.\frac{dy}{dx}\right|_{t=0} = 2$$

(27) For the Fresnel function $S(x) = \int_0^x \sin(\pi t^2/2)dt$

a) at what values of x does this function have local minima & maxima?

btc on next page

When $S' = 0$. By FTC1, $S'(x) = \sin\left(\frac{\pi x^2}{2}\right) = 0 \Leftrightarrow \frac{\pi x^2}{2} = \sin^{-1} 0$

$\sin^{-1} 0 = N\pi$, N an integer $\mathbb{Z}$.

for positive to negative (maxima), $\sin^{-1}(0) = (2n-1)\pi, n \in \mathbb{Z}$ $\cancel{\pi} x^2 = 2(2n-1)\cancel{\pi}$

maxima ⇔ $x = \sqrt{4n-2}, n \in \mathbb{Z}$

for neg to pos (minima) $\sin^{-1} 0 = 2n\pi, n \in \mathbb{Z} \rightarrow \cancel{\pi} x^2 = 2 \cdot 2n\cancel{\pi}$

minima when $x = \sqrt{4n} = 2\sqrt{n}, n \in \mathbb{Z}$

5.4 #27 b + c

Skip Lester
1/15/2013 CALCULUS II

b) On what interval is $S(x) = \int_0^x \sin(\pi t^2/2)dt$ concave upward?

When $S''(x) > 0$. $S'(x) = \sin(\pi x^2/2)\, dx$. $\rightarrow \frac{d}{dx} \sin \pi \frac{x^2}{2} = \cos \pi \frac{x^2}{2} \cdot x\pi$

$\pi x \cos(\pi \cdot x^2/2) > 0 \therefore \pi x > 0$

$= \pi x \cos \pi \frac{x^2}{2}$

$\pi x > 0$, $\cos \pi \frac{x^2}{2} > 0$ by cosine is positive on $(0, \pi/2)$

$\hookrightarrow 0 < \pi \frac{x^2}{2} < \frac{\pi}{2}$

c)

Mathematica recognized FresnelS[x]. Graphed it $\rightarrow y = .2 \Leftrightarrow \underline{x \approx .75}$

ReDo #27 here:

Evaluate the Integral by making the given substitution

1-51 odds

Skip Lester Calculus II
1/16/2013 Chapter 5.5

1. $\int e^{-x}\,dx,\ u=-x$

$\frac{du}{dx}=-1$

$dx=-du$

$=\int e^{u}(-du)$

$=-e^{u}+C$ sub.

$=\underline{-e^{-x}+C}$ $\frac{d}{dx}\Rightarrow -1\cdot[e^{-x}\cdot -1]=e^{-x}$

3. $\int x^2\sqrt{x^3+1}\,dx$ $u=x^3+1 \Rightarrow u-1=x^3$, $\sqrt[3]{u-1}=x$ $\frac{du}{dx}[x^3+1]$

$\Rightarrow 3x^2=\frac{du}{dx}$

$3x^2dx=du$

$x^2dx=\frac{du}{3}$

$=\int\sqrt{u}\;x^2dx$

$=\int\sqrt{u}\left(\frac{du}{3}\right)$

$=\frac{1}{3}\int\sqrt{u}\,du$

$=\frac{1}{3}\cdot\frac{2}{3}u^{(3/2)}$

$=\frac{2\sqrt{u^3}}{9}=\underline{\frac{2\sqrt{(x^3+1)^3}}{9}+C}$

⑤ $\int \cos^3\theta\sin\theta\,d\theta,\ u=\cos\theta$

$\frac{du}{d\theta}=-\sin\theta$

$-du=\sin\theta d\theta$

$=\int u^3(-du)$

$=-\frac{u^4}{4}+C=\underline{\frac{-\cos^4\theta}{4}+C}$

⑦ $\int x\sin(x^2)dx$ let $u=x^2$ $du=2xdx$

$=\int \underbrace{1/2\,du}_{=xdx}(\sin u)$ $=-1/2\cos x^2+C$

$=\frac{1}{2}\int\sin u\,du=\underline{\frac{1}{2}[-\cos u]+C}$

⑨ $\int(3x-2)^{20}dx$ let $u=3x-2$

$du=3dx$

$=\int u^{20}\left(\frac{du}{3}\right)$

$=\frac{1}{3}\left(\frac{u^{21}}{21}\right)+C$

$=\underline{\frac{(3x-2)^{21}}{63}+C}$

⑪ $\int\sin\pi t\,dt$ let $u=\pi t, du=\pi dt$

$=\int\sin u\frac{du}{\pi}$ $=\underline{\frac{-\cos\pi t}{\pi}+C}$

$=\frac{1}{\pi}\int\sin u\,du$

$=\frac{1}{\pi}[-\cos u]+C$

13. $\int\frac{(\ln x)^2}{x}dx$ let $u=\ln x$

$du=1/x\,dx$

$=\int u^2du$

$=u^3/3$

$=\underline{[(\ln x)^3]/3+C}$

15. $\int\frac{dx}{5-3x}$ let $u=5-3x$ $du=-3dx$

$\frac{-du}{3}=dx$

$=\int -du/3u=\frac{-1}{3}\int\frac{1}{u}du$

$=\frac{-1}{3}\ln|u|+C=\underline{\frac{-1}{3}\ln|5-3x|+C}$

17. $\int\frac{a+bx^2}{\sqrt{3ax+bx^3}}dx$ let $u=3ax+bx^3$

$du=3a+3bx^2dx$

$=3(a+bx^2)$

$=\int\frac{\frac{1}{3}du}{\sqrt{u}}$

$=\frac{1}{3}\int\frac{1}{\sqrt{u}}du\Rightarrow\frac{1}{3}[2\sqrt{u}]$

$=\boxed{\frac{2\sqrt{3ax+bx^3}}{3}+C}$

19. $\int e^x\sqrt{1+e^x}\,dx$ let $u=1+e^x$ $du=e^xdx$

$=\int\sqrt{1+e^x}\,e^xdx=\int\sqrt{u}\,du=\underline{\frac{2\sqrt{(1+e^x)^3}}{3}+C}$

21. $\int\frac{\cos x}{\sin^2x}dx$ let $u=\sin x$

$du=\cos x\,dx$

$=\int\frac{du}{u^2}=-u^{-1}+C=-\frac{1}{\sin x}+C$

$=\int u^{-2}du$ $\underline{=-\text{cosecant}\,x+C}$

Sub, relate, solve, solve, evaluate. 23-51 odd

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1/16/2013 Chapter 5.5

㉓ $\int (x^2+1)(x^3+3x)^4 dx$

Let $u = x^3+3x$ $= \int \frac{du}{3} u^4$
$du = 3x^2+3\,dx$ $= \frac{1}{3}\int u^4 du$
$du = 3(x^2+1)\,dx$
$\frac{du}{3} = x^2+1\,dx$ $= \frac{1}{3}\frac{u^5}{5}$

$$= \frac{(x^3+3x)^5}{15} + C$$

㉗ $\int \frac{dx}{\sqrt{1-x^2}\sin^{-1}x}$ let $u = \sin^{-1}x$
$du = \frac{1}{\sqrt{1-x^2}}dx$

$= \int \frac{1}{u} du$ $= \ln|\sin^{-1}x| + C$

$= \ln|u| + C$

㉛ $\int x(2x+5)^8 dx$ let $u = 2x+5$
$du = 2dx$
$x = \frac{u-5}{2}$
$dx = \frac{du}{2}$

$= \int \left(\frac{u-5}{2}\right)(u^8)\frac{du}{2}$

$= \frac{1}{4}\int (u-5)(u^8)du$

$= \frac{1}{4}\int u^9 - 5u^8 du$

$= \frac{1}{4}\left(\frac{u^{10}}{10} - \frac{5u^9}{9}\right) + C$

$$= \frac{(2x+5)^{10}}{40} - \frac{5(2x+5)^9}{36} + C$$

㊲ evaluate + graph
$\int x(x^2-1)^3 dx$ let $u = x^2-1$
$du = 2xdx$

$= \int \frac{1}{2}du(u^3) = \frac{1}{2}\int u^3 du$

$= \frac{1}{2}\left[\frac{u^4}{4}\right] = \frac{(x^2-1)^4}{8} + C$

㉕ $\int \sqrt{\cot x}\,\csc^2 x\,dx$

Let $u = \cot x$; $du = -\csc^2 x dx$

$= \int \sqrt{u}\ -du$ $= \frac{-2\sqrt{\cot x^3}}{3} + C$

$= -\int \sqrt{u}\,du$

$= -\frac{2u^{3/2}}{3} + C$ or $-\frac{2}{3}\cot x^{3/2} + C$

㉙ $\int \sec^3 x \tan x\,dx$ let $u = \sec x$
$\sec^2 x(\sec x \tan x)dx$ $du = \sec x \tan x$

$= \int u^2 du = \frac{u^3}{3} + C = \frac{\sec^3 x}{3} + C$

$\boxed{33}$ $\int \frac{\sin 2x}{1+\cos^2 x}dx = 2\int \frac{\sin x \cos x}{1+\cos^2 x}$ • double angle of Sine
$\sin 2x = 2\sin x\cos x$ •

let $u = \cos x$
$du = -\sin x dx$ $= -2\int \frac{u\,du}{1+u^2}$

$= -2\cdot\int \frac{u}{1+u^2}du$

$= -2\left[\frac{1}{2}\ln(1+u^2)\right] + C$

$$= -\ln(1+\cos^2 x) + C$$

★
㉟ $\int \frac{1+x}{1+x^2}dx$ let $u = 1+x^2$ $du = 2xdx$

$= \int \frac{1}{1+x^2}dx + \int \frac{x}{1+x^2}dx$ $= \tan^{-1}x + \frac{1}{2}\ln|u|$ ★$1+x^2$ is even★

$= \tan^{-1}x + \int \frac{du/2}{u} + C$ $= \tan^{-1}x + \frac{1}{2}\ln(1+x^2) + C$

$= \tan^{-1}x + \frac{1}{2}\int \frac{1}{u}du + C$

㊴ $\int e^{\cos x}\sin x\,dx$ let $u = \cos x$ $du = -\sin x dx$

$= \int e^u - du = -\int e^u du = -e^{\cos x} + C$

41-51 odd

Skip Lester Chapter 5.5
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(41) $\int_0^1 \cos(\pi t/2)\,dt$ let $u = \pi/2\, t$

$du = (\pi/2)\,dt$

$\frac{du}{\pi/2} = dt$

$= \int_0^{\pi/2} \cos u \left(\frac{2}{\pi} du\right)$

$= \frac{2}{\pi}\int_0^{\pi/2} \cos u\, du = \frac{2}{\pi}\left[\sin u\right]_0^{(\pi/2)}$

$= \frac{2}{\pi}\left[\sin \pi/2 - \sin 0\right]$

$= \frac{2}{\pi}[1-0] = \underline{2/\pi}$

oops sorry

47↑

$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ let $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}dx$

$= \int_1^2 e^u (2du)$ $u^2 = x,$

$2\int_1^2 e^u du \Rightarrow 2\left[e^u\right]_1^2$

$\underline{= 2\left[e^2 - e^1\right]}$

(49) $\int_{-\frac{\pi}{4}}^{\pi/4} (x^3 + x^4 \tan x)\,dx$

↑odd ↑odd

$\underline{0}$ by theorem 6

f is odd $\therefore \int_{-a}^{a} f(x)\,dx = 0$

(43) $\int_0^1 \sqrt[3]{1+7x}\,dx$ let $u = 1+7x$, $du = 7dx$

$\frac{du}{7} = dx$

$= \int_1^8 u^{(1/3)} \frac{du}{7}$

$= \frac{1}{7}\int_1^8 u^{(1/3)} du$

$= \frac{1}{7}\left[\frac{3u^{4/3}}{4}\right]_1^8 = \frac{3u^{4/3}}{28} \rightarrow \frac{3}{28}\left(8^{4/3} - 1^{4/3}\right) = \underline{\frac{45}{28}}$

(45) $\int_0^1 x^2(1+2x^3)^5 dx$ let $u = 1+2x^3$

$du = 6x^2 dx$

$\int_1^3 \frac{du}{6}(u)^5 = \frac{1}{6}\int_1^3 u^5 du = 1/6\left[\frac{u^6}{6}\right]_1^3$

$= \frac{3^6 - 1^6}{36} = \frac{729-1}{36} = \underline{\frac{182}{9}}$

$\boxed{51}$ $\int_1^2 x\sqrt{x-1}\,dx$ let $u = x-1$

$du = dx$

$x = u+1$

$\int_0^1 (u+1)\sqrt{u}\,du$

$= \int_0^1 u^{3/2} + u^{1/2} du$

$= \int_0^1 u^{3/2} du + \int_0^1 u^{1/2} du$

$= \left[\frac{2u^{5/2}}{5}\right]_0^1 + \left[\frac{2u^{3/2}}{3}\right]_0^1$

$= \frac{2}{5} + \frac{2}{3}$

$= \frac{6}{15} + \frac{10}{15} = \underline{\frac{16}{15}}$

53, 55, 59, 61, 63, 65. $\boxed{66}$

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53) $\displaystyle\int_0^1 \frac{e^z+1}{e^z+z}\,dz$

$\displaystyle= \int_0^1 \frac{e^z}{e^z+z}dz + \int_0^1 \frac{1}{e^z+z}dz$

let $u = e^z+z$, $du = e^z+1\,dz$ $\displaystyle= \int_1^{e+1} \frac{du}{u}$

$\displaystyle= \int_1^{e+1} \frac{1}{u}du = \ln|u|\Big]_1^{e+1}$

$= \ln(e+1) - \ln(1)$

$= \underline{\ln(e+1)}$

(59) $\displaystyle\int_{-2}^{2}(x+3)\sqrt{4-x^2}\,dx$

$\displaystyle= \int_{-2}^{2} x\sqrt{4-x^2}\,dx + \int_{-2}^{2} 3\sqrt{4-x^2}\,dx$

$\displaystyle\overset{\uparrow}{\int_{-a}^{a}\text{odd}} = 0 + 3\left[\text{area of a semi-circle of radius } 2\right]$

$= \underline{6\pi}$ $\quad = 3\frac{\pi r^2}{2} = \frac{3\pi 4}{2}$

(55) $\displaystyle\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$ let $u = \ln x$ $du = 1/x\,dx$

$u=4 \Leftrightarrow x = e^4$, $u = 1 \Leftrightarrow x = e$

$\displaystyle= \int_1^4 \frac{1}{\sqrt{u}}du$

$\displaystyle= \int_1^4 u^{-1/2}du = 2u^{(1/2)}\Big]_1^4$

$= 2(2) - 2 = \underline{2}$

(61) $\displaystyle\overset{a)}{\int_0^1 e^{\sqrt{x}}dx} \overset{?}{=} \overset{b)}{\int_0^1 2xe^x dx} \overset{?}{=} \overset{c)}{\int_0^{\pi/2} e^{\sin x}\sin 2x\,dx}$

a) let $u = \sqrt{x}$ $\Rightarrow \displaystyle\int_0^1 e^u(2u\,du)$ $\Rightarrow$ $\underline{\displaystyle 2\int_0^1 e^u u\,du}$

$du = \dfrac{1}{2\sqrt{x}}dx$

b) let $u = x$ $\displaystyle\int_0^1 2ue^u du = \underline{2\int_0^1 ue^u du}$

c) let $u = \sin x$ $\displaystyle= \int_0^{\pi/2} e^{\sin x}\sin 2x\,dx$

$du = \cos x\,dx$

$\displaystyle= \underline{2\int_0^1 ue^u du}$ $\displaystyle= \int_0^{\pi/2} e^{\sin x}\cdot 2\sin x\cos x$

$\displaystyle= \int_0^1 e^u(2u\,du)$

(63) an oil storage tank ruptures at time $t=0$ and oil leaks from the tank at a rate of $r(t) = 100e^{-0.01t}$ liter per minute. How much oil leaks out in the first hour.

$t = 60$

$\displaystyle\int_0^{60} r(t)dt = \int_0^{60} 100e^{-0.01t}dt$ let $u = -0.01t$, $du = -0.01dt$

$dt = -100du$

$\displaystyle= 100\int_0^{-0.6} e^u(-100\,du) = -10{,}000\left[e^u\right]_0^{-.6} = -10{,}000(e^{-.6} - e^0) = \underline{4511.9 \text{ liters}}$

65, 66

Skip Lester
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ⓒ65 $f(t) = \frac{1}{2}\sin(2\pi t/5)$ let $u = \frac{2\pi}{5}t$, $du = \frac{2\pi}{5}dt$

$V(t) = \int_0^t f(u)du = \int_0^{2\pi t/5} \frac{1}{2}\sin u\left(\frac{5du}{2\pi}\right) = \frac{5}{4\pi}\int_0^{(2\pi u)/5} \sin u = \left[\,5/4\pi\left[-\cos u\right]_0^{2\pi u/5}\right.$

$= C[F(B) - F(a)]$

$a = 0,\ b = \frac{2\pi u}{5}$

$= \frac{5}{4\pi}\left[-\cos\left(\frac{2\pi t}{5}\right) - (-1)\right] = \frac{5}{4\pi}\left[1 - \cos\left(\frac{2\pi t}{5}\right)\right]$ liters

(66) $\frac{dx}{dt} = 5000\left(1 - \frac{100}{(t+10)^2}\right)$ $\frac{\text{calculators}}{\text{week}}$ find the # of calculators produced from the beginning of the third week to the end of the fourth.

$\int_3^5 5000\left(1 - \frac{100}{(t+10)^2}\right)dt$

$= \int_3^5 \frac{5000}{(t+10)^2}dt - \int_3^5 \frac{500\,000}{(t+10)^2}dt$

let $u = (t+10)^2$
$du = 2t\,dt$
$t = \pm\sqrt{u} - 10$

$= \int_{\sqrt{13}}^{\sqrt{15}} \frac{5000}{u}\left(\frac{du}{2(\sqrt{u}-10)}\right)$

$du/2(\sqrt{u}-10)$

1-31 odd

SkipLester CAlculus II
1/18/2013 Chapter 5.6

1. Evaluate the integral $\int x^2 \ln x \, dx$ using integration by parts choosing $u = \ln x$ and $dv = x^2 dx$

$du = \frac{1}{x} dx$

$V = \frac{x^3}{3}$

$\int u dv = uv - \int v du$

$\int \ln x (x^2 dx)$

$= \ln x \left(\frac{x^3}{3}\right) - \int \left(\frac{x^3}{3}\right) \frac{1}{x} dx$

$= \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 dx$

$= \frac{x^3}{3} \ln x - \frac{1}{3} \left[\frac{x^3}{3}\right] + C$

$= \frac{1}{3} \left[x^3 \ln x - \left(\frac{x^3}{3}\right)\right] + C$

3. $\int x \cos 5x \, dx$

let $u = x$ let $dv = \cos 5x \, dx$

$du = dx$ $v = \frac{\sin 5x}{5}$

$\int u dv = uv - \int v du$

$= \frac{x}{5} \sin 5x - \int \frac{\sin 5x}{5} dx$

$= \frac{x}{5} \sin 5x - \frac{1}{5} \int \sin 5x \, dx$

$= \frac{x}{5} \sin 5x + \frac{1}{25} \cos 5x + C$

⑤ $\int r e^{r/2} dr$ let $u = r$ $dv = e^{r/2} dr$

$du = dr$ $v = 2e^{r/2}$

$\int u dv = uv - \int v du$

$\int r e^{r/2} = r(2e^{r/2}) - \int 2e^{r/2} dr$

$= 2re^{r/2} - 2\int e^{r/2} dr$

$= 2re^{r/2} - 2[2e^{r/2}] + C$

$= 2re^{r/2} - 4e^{r/2} + C$

⑦ $\int x^2 \sin \pi x \, dx$ let $u = x^2$

$du = 2x dx$

$dv = \sin \pi x \, dx$

$V = \frac{-\cos \pi x}{\pi} + C$

$\int u dv =$
$uv - \int v du$

$= \frac{-x^2 \cos \pi x}{\pi} - \int -\frac{\cos \pi x}{\pi} (2x dx)$

let $u = x$, $du = dx$

$dv = \cos \pi x \, dx$

$V = \frac{\sin \pi x}{\pi}$

$\frac{-x^2 \cos \pi x}{\pi} + \frac{2}{\pi} \int x \cos(\pi x) dx;$

$= || + 2/\pi \left[\frac{x}{\pi} \sin \pi x - \int \frac{\sin \pi x}{\pi} dx \right]$

$= \left[\frac{-\cos \pi x}{\pi^2} + c \right]$

$= -\frac{x^2 \cos \pi x}{\pi} + \frac{2}{\pi} \left[\frac{x}{\pi} \sin \pi x + \frac{\cos \pi x}{\pi^2} \right] + C$

$= \frac{-x^2 \cos \pi x}{\pi} + \frac{2x \sin \pi x}{\pi^2} + \frac{2 \cos \pi x}{\pi^3} + C$

⑨ $\int \ln \sqrt[3]{x} \, dx = \int \ln x^{1/3} dx$

$= \int \frac{1}{3} \ln x dx$

$= \frac{1}{3} \int \ln x \, dx$

let $u = \ln x$ $dv = dx$

$du = \frac{1}{x} dx$ $v = x$

$u \cdot v - \int v du \rightarrow$

$x \ln x - \int \frac{x}{x} dx$

$\frac{1}{3} [x \ln x - x + C] = \frac{1}{3} x \ln x - \frac{1}{3} x + C$

$= x \ln x^{(1/3)} - \frac{1}{3} x + C$

$= x \ln \sqrt[3]{x} - 1/3 x + C$

Skip Lester Chapter 5.6
1/19/2013 Solowan 11:00 am

11-31 odd

*
11. $\int \arctan 4t\,dt$ let $u = \arctan 4t$, $dv = dt$

$du = \dfrac{4}{1+(4t)^2}$ $v = t$

$\int u\,dv = uv - \int v\,du$

$$= t\arctan t - \int \frac{4t}{(1+16t^2)}dt$$

$$= t\arctan t - 4\int \frac{t}{1+16t^2}dt$$

Let $g = 1+16t^2$

$dg = 32t\,dt$

$$= t\arctan t - \frac{1}{8}\int \frac{1}{g}dg$$

$$= \underline{t\arctan t - 1/8\ln(1+16t^2) + C}$$

*
⑬ $\int \underline{e^{2\theta}\sin 3\theta\,d\theta}$ let $u = \sin 3\theta$, let $dv = e^{2\theta}d\theta$

$du = 3\cos 3\theta\,d\theta$, $v = \dfrac{e^{2\theta}}{2} + C$

$\int u\,dv = uv - \int v\,du$

$$\Rightarrow \frac{e^{2\theta}\sin 3\theta}{2} - \int \frac{3e^{2\theta}\cos 3\theta\,d\theta}{2}$$

$$= \frac{e^{2\theta}\sin 3\theta}{2} - \frac{3}{2}\int \underbrace{e^{2\theta}\cos(3\theta)\,d\theta}$$

— Regarding $\int e^{2\theta}\cos 3\theta\,d\theta$, —

— Let $\omega = \cos 3\theta$, $dx = e^{2\theta}d\theta$ —

— $d\omega = -3\sin 3\theta\,d\theta$ $x = [e^{2\theta}/2] + C$ —

— $\int \omega\,dx = \omega x - \int x\,d\omega$ —

$$\Rightarrow \overbrace{\frac{e^{2\theta}\cos 3\theta}{2} + \frac{3}{2}\int \sin 3\theta\, e^{2\theta}\,d\theta}$$

$$S = \frac{e^{2\theta}\sin 3\theta}{2} - \frac{3e^{2\theta}\cos 3\theta}{4} - \frac{9}{4}\int \underline{e^{2\theta}\sin 3\theta\,d\theta}$$

$+9/4S$ Let $S = \int e^{2\theta}\sin 3\theta\,d\theta$ $+ 9/4\,S$

$$\frac{13}{4}S = \frac{e^{2\theta}\sin 3\theta}{2} - \frac{3e^{2\theta}\cos 3\theta}{4} + C$$

$$S = \frac{4}{13}e^{2\theta}\left(\frac{1}{2}\sin 3\theta - \frac{3}{4}\cos 3\theta\right) + C$$

⑮ $\int_0^{\pi} t\sin 3t\,dt$ let $u = t$, $dv = \sin 3t\,dt$

$du = dt$, $v = -\dfrac{\cos 3t}{3}$

$\int u\,dv = uv - \int v\,du$

$$= \left[-\frac{t\cos 3t}{3}\right]_0^{\pi} - \int_0^{\pi} \frac{-\cos 3t}{3}dt$$

$$= \left[-\frac{t\cos 3t}{3}\right]_0^{\pi} + \frac{1}{3}\int_0^{\pi}\cos 3t\,dt$$

$$= \left[-\frac{t\cos 3t}{3}\right]_0^{\pi} + \frac{1}{3}\left[\frac{\sin 3t}{3}\right]_0^{\pi}$$

$$= \frac{\pi}{3} - 0 + \left[\frac{1}{9}\left(\overset{\sin 9\pi}{0} - \overset{\sin 0}{0}\right)\right]$$

$$= \underline{\pi/3}$$

⑰ $\int_1^2 \dfrac{\ln x}{x^2}dx$ Let $u = \ln x$ let $dv = x^{-2}dx$

$du = \dfrac{1}{x}dx$ $v = -x^{-1}$

$$= -\frac{\ln x}{x} - \int -\frac{1}{x}\left(\frac{1}{x}\right)dx \qquad \frac{d}{dx}x^{-1} = -x^{-2}$$

$$= \left[\frac{-\ln x}{x}\right]_1^2 + \int \frac{dx}{x^2} \qquad \int x^{-2}dx \Rightarrow -x^{-1}$$

$$= \left[\frac{-\ln x}{x}\right]_1^2 + \left[\frac{-1}{x}\right]_1^2 = (-\ln 2/2) + (\ln 1) + [(-1/2) + 1]$$

$$= \underline{\frac{-\ln 2}{2} + \frac{1}{2}}$$

⑲ $\int_0^1 y/e^{2y}\,dy$ let $u = y$; $dv\ dy/e^{2y} = e^{-2y}dy$

$du = dy$ $v = \frac{-1}{2}e^{-2y}$

$\int u\,dv = uv - \int v\,du$

$$= \frac{-y}{2\cdot e^{2y}} - \int \frac{-dy}{2e^{2y}} \qquad = \left[\frac{-1}{2}ye^{-2y}\right]_0^1 + \frac{1}{2}\left[\frac{-1}{2}e^{-2y}\right]_0^1$$

$$= -\frac{1}{2}ye^{-2y} + \frac{1}{2}\int e^{-2y}dy \qquad = \left[\frac{-1}{2}(1)e^{-2} - 0\right] - \frac{1}{4}\left[e^{-2} - 1\right]$$

$$= -\frac{1}{2e^2} - \frac{1}{4e^2} + \frac{1}{4}$$

$$= \underline{\frac{-3}{4e^2} + \frac{1}{4}}$$

Integration by Parts
21-31 odd $\int u\,dv = uv - \int v\,du$

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㉑ $\int_0^{1/2} \arccos x\,dx$ let $u = \cos^{-1} x$, $dv = dx$

$du = \frac{-1}{\sqrt{1-x^2}}dx$ $v = x$

$\int u\,dv = u\cdot v - \int v\,du$

$= \left[x \arccos x\right]_0^{1/2} - \int_0^{1/2} \frac{-x}{\sqrt{1-x^2}}dx$

$= \frac{1}{2}\cos^{-1}\left(\frac{1}{2}\right) - 0 + \int_1^{3/4} w^{-1/2}\left(-\frac{1}{2}dw\right)$

$= \left(\frac{1}{2}\cdot\frac{\pi}{3}\right) + \frac{1}{2}\int_{3/4}^{1} w^{-1/2}dw$

$= \frac{\pi}{6} + \frac{1}{\cancel{2}}\left[\cancel{2}\sqrt{w}\right]_{3/4}^{1}$

$= \frac{\pi}{6} + \left[\sqrt{1} - \sqrt{3/4}\right]$

$= \frac{\pi}{6} + 1 - \frac{\sqrt{3}}{2}$

let $w = 1-x^2$
$dw = 2x\,dx$
$-x^2 = w - 1$
$x^2 = -w + 1$
$x = \pm\sqrt{1-w}$
$\frac{1}{2} = +\sqrt{1-w}$
$\frac{1}{4} = 1-w$
$\frac{-3}{4} = -w$
$0 = \sqrt{1-w}$
$1 = w$

㉕ First, make a substitution, then use Integration by Parts to evaluate the integral.

$\int \cos\sqrt{x}\,dx$ Let $y = \sqrt{x}$ $dy = \frac{1}{2\sqrt{x}}dx$

$= \int \cos y\,(2y\,dy)$ $\therefore 2y\,dy = dx$

$= 2\int y\cos y\,dy$ Let $u = y$, $dv = \cos y\,dy$
$du = dy$ $v = \sin y$

$= 2\left[(y\sin y) - \int \sin y\,dy\right] = 2y\sin y + 2\cos y + C$

$= 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C$

㉓ $\int_1^2 (\ln x)^2 dx$ let $u = (\ln x)^2$, $dv = dx$

$du = 2\cdot\ln x\cdot(1/x)dx$ $v = x$
$= \frac{2}{x}\ln x\,dx$

$\int_1^2 u\,dv = [uv]_1^2 - \int_1^2 v\,du$

$= \left[x(\ln x)^2\right]_1^2 - \int_1^2 \cancel{x}\cdot\frac{2}{\cancel{x}}\ln x\,dx$

$= \left[x(\ln x)^2\right]_1^2 - 2\int_1^2 \ln x\,dx$

let $u = \ln x$, $dv = dx$
$du = \frac{1}{x}dx$ $v = x$
$\int u\,dv = u\cdot v - \int v\,du$
$= x\ln x - \int_1^2 \frac{x}{x}dx$
$= [x\ln x - x]_1^2$

$= [x\ln x - 2x\ln x + 2x]_1^2$

$= [2(\ln 2)^2 - 4\ln 2 + 4]$
$- [\ln 1 - 2\ln 1 + 2]$

$= 2(\ln 2)^2 - 4\ln 2 + 2$

㉗ $\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3\cos(\theta^2)\,d\theta$ Let $u = \theta^2$, $du = 2\theta\,d\theta$
$u\,du = 2\theta^3 d\theta$

$= \int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^2\cos(\theta^2)\left(\frac{1}{2}(2\theta\,d\theta)\right)$

$= \frac{1}{2}\int_{\pi/2}^{\pi} u\cos u\,du$ let $U = u$, $dv = \cos u\,du$
$dU = du$ $v = \sin u$

$\int U\,dv = U\cdot v - \int v\,dU$

$\frac{1}{2}\left[u\sin u - \int \sin u\,du\right]$

$= \frac{1}{2}\left[u\sin u + \cos u\right]_{\frac{\pi}{2}}^{\pi} = \frac{1}{2}(0-1) - \left[\frac{1}{2}\left(\frac{\pi}{2} - 0\right)\right]$

$= -1/2 - \pi/4$

Skip Lester Calculus II
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29, 31

29. $\int x \ln(1+x)\,dx$ Let $y=(1+x)$, $dy=dx$
$x=y-1$

$\int (y-1)\ln y\,dy$ let $u=\ln y$ let $dv=(y-1)dy$

$\int u\,dv = u\cdot v - \int v\,du$ $du=\frac{1}{y}dy$ $v=\frac{y^2}{2}-y$

$=\ln y\left(\frac{y^2}{2}-y\right)-\int\left(\frac{y^2}{2}-y\right)\left(\frac{1}{y}dy\right)$

$=\frac{y^2\ln y}{2}-y\ln y-\int\left(\frac{y}{2}-1\right)dy$

$=\frac{y^2\ln y}{2}-y\ln y-\left[\frac{y^2}{4}-y\right]+C$

$=\frac{1}{2}y(y-2)\ln y-\left[\frac{1}{4}y(y-4)\right]+C$

$=\frac{1}{2}(x+1)(x+1-2)\ln(x+1)-\frac{1}{4}(x+1)^2+(x+1)+C$

$=\frac{1}{2}(x-1)^2\ln(x+1)-\frac{x^2}{4}-\frac{2x}{4}-\frac{1}{4}+x+1+C$

$=\frac{(x-1)^2\ln(x+1)}{2}-\frac{x^2}{4}+\frac{x}{2}+\frac{3}{4}+C$

Evaluate + Graph

31. $\int xe^{-2x}dx$ let $u=x$, $dv=e^{-2x}dx$
$du=dx$ $v=\frac{e^{-2x}}{-2}$

$\int u\,dv=u\cdot v-\int v\,du$

$=\frac{xe^{-2x}}{-2}+\int\frac{e^{-2x}}{2}dx$

$=\frac{xe^{-2x}}{-2}+\frac{1}{2}\left[\frac{e^{-2x}}{-2}\right]+C$

$=\frac{-1}{2}\left[xe^{-2x}+\frac{1}{2}\left(e^{-2x}\right)\right]+C$

Chapter 5.6

26-32 evens 41;45. Skip Lester Calculus II 1/22/2013

26) $\int t^3 e^{-t^2} dt$ Let $\omega = t^2, d\omega = 2t\,dt$

$(\frac{1}{2}d\omega = t\,dt)$

$= \int \omega e^{-\omega}(\frac{1}{2}d\omega)$

$= \frac{1}{2}\int \omega e^{-\omega} d\omega$ let $u=\omega$ $dv = e^{-\omega}d\omega$

$du = d\omega$ $v = -e^{-\omega}$

$\int u\,dv = u \cdot v - \int v\,du$

$= \frac{1}{2}(-\omega e^{-\omega} - \int -e^{-\omega} d\omega)$

$= \frac{1}{2}(-\omega e^{-\omega} - e^{-\omega}) + C$

$= \frac{-t^2 e^{-t^2} - e^{-t^2}}{2} + C$

$= \frac{-e^{-t^2}(t^2+1)}{2} + C$

28) $\int_0^{\pi} e^{\cos t} \sin 2t\,dt$ let $u = \cos t$ $du = -\sin t$

$= \int_0^{\pi} e^{\cos t} 2\sin t \cos t\,dt$ $= 2\int_1^0 e^u u(-1 \cdot du)$

$= 2\int_0^{\pi} e^{\cos t} \sin t \cos t\,dt$ $= -2\int_1^0 e^u u\,du$

$\int f\,dg = fg - \int g\,df$ let $f = u$ $dg = e^u du$

$df = du$ $g = e^u$

$= (ue^u - \int e^u du)(-2)$

$= -2ue^u + 2e^u \Big|_1^0$ $= -2\cos t e^{\cos t} + 2e^{\cos t}$

$= -2e^{\cos t}(\cos t - 1)$

$F(\pi) - F(0)$

$= -2e^{\cos\pi}\cos\pi + 2e^{\cos\pi} - 2e^{\cos 0}\cos 0 + 2e^1$

$-2e^{-1}(-1) + 2e^{-1} - 0$

$2e^{-1} + 2e^{-1} = 4e^{-1}$

$\boxed{= 4/e}$

30) $\int \sin(\ln x)dx$ let $u = \ln x$ $du = \frac{1}{x}dx$

$x = e^u$

$du = \frac{dx}{e^u}$

$dx = e^u du$

$= \int \sin(u) e^u du$

$\int f\,dg = fg - \int g\,df$

Let $f = e^u$ $dg = \sin u\,du$ $\Rightarrow$

$df = e^u du$ $g = -\cos u$

$\Rightarrow -e^u \cos u - \int -e^u \cos u\,du$

$= -e^u \cos u + \int e^u \cos u\,du$

$f = e^u$ $dg = \cos u\,du$

$df = e^u du$ $g = \sin u$

$\int e^u \cos u\,du = e^u \sin u - \int e^u \sin u\,du$

$\therefore \int \sin(u) e^u du = -e^u \cos u + e^u \sin u - \int \sin u e^u du$

let $\omega = \int \sin(u) e^u du$

$\omega = -e^u \cos u + e^u \sin u - \omega$

$+\omega$ $+\omega$

$2\omega = e^u(\sin u - \cos u)$

$\therefore \int \sin u e^u du = \frac{e^u(\sin u - \cos u)}{2} + C$

$= \frac{e^{\ln x}(\sin(\ln x) - \cos(\ln x))}{2} + C$

$(\frac{x}{2})(\sin(\ln x) - \cos(\ln x)) + C$

32) $\int x^{3/2} \ln x\,dx$ $\int u\,dv = uv - \int v\,du$

let $u = \ln x$ $dv = x^{3/2}dx$

$du = \frac{1}{x}dx$ $v = \frac{2}{5}x^{5/2}$

$= \frac{2x^{5/2}\ln x}{5} - \int \frac{2}{5}x^{5/2}(\frac{1}{x})dx$

$= \frac{2x^{5/2}\ln x}{5} - \frac{2}{5}\int x^{3/2}dx$

$= \frac{2x^{5/2}\ln x}{5} - \frac{2}{5}(\frac{2x^{5/2}}{5}) + C$

$= \frac{2x^{5/2}\ln x}{5} - \frac{4x^{5/2}}{25} + C$

41) $\int (\ln x)^3 dx$

Chapter 5.6 39-45 Skip ~~later~~

39,40,41-45

39) $\int (\ln x)^n dx = x(\ln x)^n - n\int (\ln x)^{n-1} dx$

let $u = (\ln x)^n$ $dv = dx$

$v = x$

$du = n(\ln x)^{n-1}\left(\frac{1}{x}\right)dx$

$du = n(\ln x)^{n-1}\frac{dx}{x}$

$\int u dv = u \cdot v - \int v du$

$= x(\ln x)^n - \int x\left(\frac{n(\ln x)^{n-1} dx}{x}\right)$

$= \underline{x(\ln x)^n - n\int (\ln x)^{n-1} dx}$ □

41) $\int (\ln x)^3 dx$ using ⑶⑼

$= x(\ln x)^3 - 3\int (\ln x)^2 dx$

$= x(\ln x)^3 - 3\left[x(\ln x)^2 - 2\int (\ln x)^1 dx\right]$

$= x(\ln x)^3 - 3x\ln x^2 + 6\left[x\ln x - \int (\ln x)^0 dx\right]$

$= \underline{x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x - 6x + C}$

40) $\int x^n e^x dx = x^n e^x - n\int x^{n-1} e^x dx$

$\int u dv = \quad uv - \int v du$

Let $u = x^n$, $dv = e^x dx$

$du = nx^{(n-1)} dx$ $v = e^x$

$= x^n e^x - \int e^x (nx^{n-1}) dx$

$= \underline{x^n e^x - n\int x^{n-1} e^x dx}$

42) $\int x^4 e^x dx$ using ⑷⑽

Let $u = x^4$, $dv = e^x dx$ $\int u dv = u \cdot v - \int v du$

$du = 4x^3 dx$, $v = e^x$

$= x^4 e^x - 4\int x^3 e^x dx$

$= x^4 e^x - 4(x^3 e^x - 3\int x^2 e^x dx)$

$= x^4 e^x - 4x^3 e^x + 12(x^2 e^x - 2\int x e^x dx)$

$= x^4 e^x - 4x^3 e^x + 12x^2 e^x - 24(xe^x - \int e^x dx)$

$= \underline{x^4 e^x - 4x^3 e^x + 12x^2 e^x - 24xe^x + 24e^x + C}$

$= \underline{e^x(x^4 - 4x^3 + 12x^2 - 24x + 24) + C}$

(43) A particle that moves along a straight line has velocity $v(t) = t^2 e^{-t}$ m/s after t seconds. How far will it travel during the first t seconds?

$r(t) = \int v(t) = \int t^2 e^{-t} dt$ $u = t^2$ $dv = e^{-t} dt$

$du = 2t dt$ $v = -e^{-t}$

$\int u dv = u \cdot v - \int v du$

$u = t$ $dv = e^{-t} dt$

$du = dt$ $v = -e^{-t}$

$= -t^2 e^{-t} - \int -e^{-t} 2t dt$

$= -t^2 e^{-t} + 2\int t e^{-t} dt$

$= -t^2 e^{-t} + 2(-te^{-t} - \int -e^{-t} dt)$

$= (-t^2 e^{-t} - 2te^{-t} - 2e^{-t})$ meters

$= \underline{e^{-t}(-t^2 - 2t - 2) \text{ meters}}$

44) A rocket accelerates by burning its onboard fuel, so its mass decreases with time.

Suppose the initial mass of the rocket, with fuel, is m, the fuel is consumed at rate r, and the exhaust gases are ejected at constant velocity v_e, relative to the rocket.

A model for the velocity at time t is given by: $v(t) = -gt - v_e \ln\left(\frac{m - rt}{m}\right)$

where $g = 9.8 \text{ m/s}^2$, $m = 30{,}000$ kg, $r = 160$ kg/s and $v_e = 3000$ m/s

Find the height of the rocket at $t = 60$ s

$h(t) = \int -gt - v_e \ln\left(\frac{m - rt}{m}\right) dt$

let $u = \frac{m - rt}{m}$

$du = \frac{-r}{m} dt$

$dt = \frac{-m}{r} du$

$= \int -gt - v_e \ln(u)\left(\frac{-m}{r}\right) du$

$= \frac{-gt^2}{2} + \frac{mv_e}{r}\int \ln u \, du$

$= \frac{-gt^2}{2} + \frac{mv_e}{r}\left[u\ln u - u\right]$

44 $= \frac{mv_e}{r}\left[\left(\frac{m - rt}{m}\right)\ln\left(\frac{m - rt}{m}\right) - \left(\frac{m - rt}{m}\right)\right] - \frac{gt^2}{2}$

$=$

Chapter 5.6: Int. by Parts
44, con't, 45

Skip Lotter Calculus II 1/22/2013

44) from last page, $v(t) = -gt - v_e \ln\left(\frac{m-rt}{m}\right)$. $h(t) = \int -gt - v_e \ln\left(\frac{m-rt}{m}\right) dt$

45) Suppose that $f(1)=2$, $f(4)=7$, $f'(1)=5$, $f'(4)=3$, and f'' is continuous. Find the value of $\int_1^4 x f''(x)\,dx$

Let $u=x$ $dv=f''(x)dx$
$du=dx$ $v=f'(x)$

$\int u\,dv = u \cdot v - \int v\,du$

$= \left[x f'(x)\right]_1^4 - \int_1^4 f'(x)\,dx$

$=(4 \cdot 3)-(1 \cdot 5)-(7-2)$

$12-5-5=\underline{2}$

Appendix G
Write in form

1) a) $\frac{2x}{(x+3)(3x+1)} = \underline{\frac{A}{x+3} + \frac{B}{3x+1}}$

b) $\frac{1}{x^3+2x^2+x} = \frac{1}{x(x^2+2x+1)}$

$\underline{\frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}} = \frac{1}{x(x+1)(x+1)}$

3) a) $\frac{x^4+1}{x^5+4x^3} = \frac{x^4+1}{x^3(x^2+4)} \Rightarrow \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{(x^2+4)}$

- Is degree of numerator < degree of denominator?

5) a) $\frac{x^4}{x^4-1} = \frac{(x^4-1)+1}{x^4-1} = 1\,\frac{1}{x^4-1}$

$=1\,\frac{1}{(x-1)(x+1)(x^2+1)} \rightarrow \underline{\frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}}$

b) $\frac{1}{(x^2-9)^2} = \frac{1}{(x-3)(x+3)(x-3)(x+3)}$

$\underline{\frac{A}{(x-3)} + \frac{B}{(x-3)^2} + \frac{C}{(x+3)} + \frac{D}{(x+3)^2}}$

b) $\frac{t^4+t^2+1}{(t^2+1)(t^2+4)^2} = \frac{At+B}{t^2+1} + \frac{Ct+D}{(t^2+4)} + \frac{Et+F}{(t^2+4)^2}$

Appendix G 9-33 odd +43 Skip Lester 1/23/2013

9) $\int \frac{x-9}{(x+5)(x-2)} dx$

$\Rightarrow \int \frac{x-9}{(x+5)(x-2)} = \int \underbrace{\left(\frac{A}{(x+5)} + \frac{B}{(x-2)}\right)}$

$x-9 = A(x-2) + B(x+5)$

$x-9 = Ax - A2 + Bx + B5$

$x^0 = -9$
$x^1 = 1$

$-9 = -2A + 5B \quad 1 = A + B$
$-9 = -2(1-B) + 5B \quad A = 1 - B$
$-9 = -2 + 2B + 5B \quad A = 1-(-1)$
$+2 \quad +2 \quad B = -1 \quad A = 2$
$-7 = 7B$
$/7 \quad /7$

Pro-tip: $\int \frac{1}{u} du = \ln|u| + c$

$\int \frac{2}{(x+5)} + \frac{-1}{x-2} dx$

$= 2\ln|x+5| - \ln|x-2| + C$

11) $\int_2^3 \frac{1}{x^2-1} dx = \int_2^3 \frac{1}{(x+1)(x-1)}$

$\int_2^3 \frac{1}{(x+1)(x-1)} dx = \int \frac{A}{(x+1)} + \frac{B}{(x-1)} dx$

$\int_2^3 1 dx = \int_2^3 A(x-1) + B(x+1)$

$\int_2^3 1 dx = \int_2^3 Ax - 1A + Bx + 1B \, dx \quad x^0 = 1 \quad x^1 = 0$

$1 = -1A + 1B \quad 0 = A + B$
$B = 1 + A \quad 0 = A + (1+A)$
$0 = 2A + 1$
$-1 = 2A$
$A = -1/2 \quad B = 1/2$

$\int \frac{-1/2}{(x+1)} + \frac{1/2}{(x-1)}$

$= \left[\frac{-1}{2} \ln|x+1| + \frac{1}{2} \ln|x-1|\right]_2^3$

$= \frac{-1}{2} \ln 4 + \frac{1}{2} \ln 2 + \frac{1}{2} \ln 3 - \frac{1}{2} \ln 1$

$= \frac{1}{2}\left[-\ln 4 + \ln 2 + \ln 3\right]$

⑬ $\int \frac{ax}{x^2 - bx} dx = \int \frac{a\cancel{x}}{\cancel{x}(x-b)} dx$

$= \int \frac{a}{x-b} dx = a\ln|x-b| + C$

⑮ $\int_3^4 \frac{x^3 - 2x^2 - 4}{x^3 - 2x^2} dx$ long division

$x^3 - 2x^2 \overline{\big) x^3 - 2x^2 - 4}$ quotient 1 $= 1 + \frac{-4}{x^3 - 2x^2}$
$-x^3 + 2x^2)$
$0 - 4$

$= \int_3^4 1 + \frac{-4}{x^3 - 2x^2} dx = \int_3^4 1 dx + \int_3^4 \frac{-4}{x^2(x-2)} dx$

$\int_3^4 \frac{-4}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-2)}$ L.C.D.

$x^0 = -2B \quad -4 = Ax(x-2) + B(x-2) + C(x^2)$
$-4 = Ax^2 - 2Ax + Bx - 2B + Cx^2$

$x^1 = -2A + B$
$x^2 = A + C$

$-4 = -2B \quad 0 = -2A + B \quad 0 = A + C$
$/-2 \quad /-2 \quad 0 = -2A + 2 \quad 0 = 1 + C$
$2 = B \quad -2 = -2A \quad C = -1$
$A = 1$

$\overset{\ln|x|}{} \quad \overset{-2x^{-1}}{}$

$= \int_3^4 \left(\frac{1}{x} + \frac{2}{x^2} - \frac{1}{x-2} + 1\right) dx$

$= \left[\ln|x| - \frac{2}{x} - \ln|x-2| + x\right]_3^4$

$= [\ln 4 - 2/4 - \ln 2 + 4] - [\ln 3 - 2/3 - \ln 1 + 3]$
$= \ln 4 - \ln 3 - \ln 2 + 4 - 3/6 + 4/6 - 3$

$= \ln 4 - \ln 3 - \ln 2 + 1/6 + 1$

$\ln(4/3) - \ln 2 + 7/6$

$= \ln(2/3) + 7/6$

Appendix G
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17) $\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)} dy$

$\Rightarrow \int_1^2 \frac{A}{y} + \frac{B}{y+2} + \frac{C}{y-3} = \int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)} dy$

$= A(y+2)(y-3) + B(y)(y-3) + C(y)(y+2) = 4y^2-7y-12$

$= Ay^2 - Ay - 6A + By^2 - 3By + Cy^2 + 2Cy$

$y^0: -12 = -6A \quad \underline{A=2}$

$y^1: -7 = -A - 3B + 2C \rightarrow -7 = -2 - 3B + 2C$

$+2 \quad +2$

$y^2: 4 = A + B + C \qquad -5 = -3B + 2C$

$-5 = -3B + 2(2-B)$

$4 = 2 + B + C \qquad -5 = -3B + 4 - 2B$

$2 = B + C \qquad -4 \quad -4$

$2 - B = C \qquad -9 = -5B$

$\underline{C = 1/5} \qquad \underline{B = 9/5}$

$= \int_1^2 \frac{2}{y} + \frac{9/5}{y+2} + \frac{1/5}{y-3} dy$

$= \left[2\ln|y| + 9/5\ln|y+2| + 1/5\ln|y-3|\right]_1^2$

$= [2\ln 2 + 9/5\ln 4 + 1/5(0)] - [2(0) + 9/5\ln 3 + 1/5\ln 2]$

$= 9/5\ln 4 - 9/5\ln 3 + 9/5\ln 2$

$= 9/5\ln(4/3) + 9/5\ln 2$

$= \underline{9/5\ln(8/3)}$

19) $\int \frac{1}{(x+5)^2(x-1)} dx = \int \frac{A}{(x-1)} + \frac{B}{(x+5)} + \frac{C}{(x+5)^2} dx$

$1 = A(x+5)^2 + B(x+5)(x-1) + C(x-1)$

$A(x^2+10x+25) + Bx + 5B(x-1) + Cx - 1C$

$Bx^2 - Bx + 5Bx - 5B$

$1 = Ax^2 + 10Ax + 25A + Bx^2 + 4Bx - 5B + Cx - 1C$

$x^0: 1 = 25A - 5B - C$

$x^1: 0 = 10A + 4B + C$

$x^2: 0 = A + B$

$A = -B$

$-A = B$

$1 = 25A + 5A - C \qquad 10A + 4B + C = 0 \qquad 0 = 10/36 - 4/36 + C$

$1 = 30A - C \qquad 10A - 4A + 30A - 1 = 0 \qquad 0 = 6/36 + C$

$1 - 30A = -C \qquad 36A = 1$

$C = 30A - 1 \qquad \underline{A = 1/36} \therefore \underline{B = -1/36} \qquad \underline{C = -6/36 = -1/6}$

$\Rightarrow \int \frac{1/36}{(x-1)} - \frac{1/36}{(x+5)} - \frac{1/6}{(x+5)^2} \Rightarrow \underline{\frac{1}{36}\ln|x-1| - \frac{1}{36}\ln|x+5| + \frac{1}{6(x+5)} + C}$

$\frac{1}{6(x+5)^2}$ let $u = (x+5)$

$\int \frac{-1}{6u^2} = \frac{1}{6u}$

Appendix G
21-33 odd

Skip Lester 1/23/2013

21) $\int \frac{5x^2+3x-2}{x^3+2x^2}dx = \int \frac{5x^2+3x-2}{x^2(x+2)}dx$

$\Rightarrow \int \frac{5x^2+3x-2}{x^2(x+2)}dx = \int \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$

Clear fraction with L.C.D.

$5x^2+3x-2 = Ax(x+2) + B(x+2) + C(x^2)$

$5x^2+3x-2 = Ax^2+2Ax+Bx+2B+Cx^2$

$x^0 \Rightarrow -2=2B \quad B=-1$

$x^1 \to 3=2A+B$

$x^2 \to 5=A+C \quad 5=2+C \quad C=3$

$3=2A-1$
$+1 \quad +1$
$4=2A$
$A=2$

$A=2, B=-1, C=3$

$\to \int \frac{2}{x} - \frac{1}{x^2} + \frac{3}{x+2} = \underline{2\ln|x| + 1/x + 3\ln|x+2| + C}$

23) $\int \frac{10}{(x-1)(x^2+9)}dx = \int \frac{A}{x-1} + \frac{Bx+C}{(x^2+9)}dx$

$10 = A(x^2+9) + Bx+C(x-1)$

$10 = Ax^2+9A \quad +Bx^2-Bx+Cx-1C$

$x^0 \Rightarrow 10=9A-1C \to$
$x^1 \Rightarrow 0=C-B \quad B=C$
$x^2 \Rightarrow 0=A+B \quad \underline{B=-A}$

$10=-9B-B$
$10=-10B$
$B=-1, C=-1, A=1$

$= \int \frac{1}{x-1} + \frac{-x-1}{(x^2+9)}dx = \int \frac{1}{x-1} - \frac{x}{(x^2+9)} - \frac{1}{(x^2+9)} = \boxed{\ln|x| - \frac{1}{2}\ln(x^2+9) - \frac{1}{3}\tan^{-1}\frac{x}{3} + C}$

let $u=x^2+9$
$du=2x\,dx$ $\int \frac{x}{(x^2+9)}dx = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln(x^2+9)$

25) $\int \frac{x^3+x^2+2x+1\ dx}{(x^2+1)(x^2+2)}$

$= \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+2)}$

$x^3+x^2+2x+1 = [(Ax+B)(x^2+2)] + [(Cx+D)(x^2+1)]$

$x^3+x^2+2x+1 = Ax^3+2Ax+Bx^2+2B+Cx^3+Cx+Dx^2+D$

$= \int \frac{x}{x^2+1} + \frac{1}{(x^2+2)}dx = \underline{\frac{1}{2}\ln|x^2+1| + \frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}} + C}$

$x^3: 1=A+C \quad C=1-A$
$x^2: 1=B+D \quad 1=B+(1-2B)$
$x^1: 2=2A+C \quad 1=B+1 \quad \underline{B=0} \quad 2=2A+(1-A)$
$x^0: 1=2B+D \quad D=1-2B \quad 2=A+1$
$\underline{D=1}$ $\quad \underline{A=1}$
$\underline{C=0}$

$\int \frac{1}{(x^2+2)} = \int \frac{1}{2(\frac{x^2}{2}+1)}$

let $u=\frac{x^2}{2}$ $du=x\,dx$

$\frac{du}{2x}=dx$

27) $\int \frac{x+4}{x^2+2x+5}dx = \int \frac{x+1}{x^2+2x+5}dx + \int \frac{3}{x^2+2x+5}dx = \frac{1}{2}\int \frac{2x+2\,dx}{x^2+2x+5} + \int \frac{3dx}{x^2+2x+5}$

let $u=x^2+2x+5$
$du=2x+2dx$ $\int \frac{du}{u} = \ln u$

$= \frac{1}{2}\ln|x^2+2x+5| + \int \frac{3\,dx}{x^2+2x+1+4}$

$= \frac{1}{2}\ln|x^2+2x+5| + \int \frac{3dx}{(x+1)^2+4}$ let $2u=x+1$, $2du=dx$ $= 3\int \frac{2du}{(2u)^2+4} = 3\int \frac{2du}{4u^2+4} = 3\int \frac{2du}{4(u^2+1)}$

$= 3/2 \int \frac{du}{u^2+1}$

$= \underline{\frac{1}{2}\ln|x^2+2x+5| + \frac{3}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + C}$

$= \int \frac{1dx}{2(\frac{x}{\sqrt{2}})^2+1} = \frac{1}{2}\int \frac{1dx}{(\frac{x}{\sqrt{2}})^2+1}$

$\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$

let $v=\frac{x}{\sqrt{2}}$
$dv=\frac{1}{\sqrt{2}}dx$

$= \frac{1}{2}\int \frac{dv}{\sqrt{2}(v^2+1)} = \frac{1}{\sqrt{2}}\int \frac{1}{v^2+1}dv$

$= \frac{1}{\sqrt{2}}\tan^{-1}v$

Appendix G 29, 31, 33, 43

Skip LesteR 1/23/2013

29) $\int \frac{1\,dx}{x^3-1} = \int \frac{dx}{(x-1)(x^2+x+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+x+1)}$

difference of two cubes $1 = A(x^2+x+1) + [(Bx+C)(x-1)]$

$\Rightarrow 1 = Ax^2+Ax+A-Bx+Bx^2+Cx-C$

$x^2: 0=A+B \quad -A=B \quad B=-1/3$

$x^1: 0=A-B+C \quad A-(-A)+(A-1)=0 \quad 3A=1$

$x^0: 1=A-C \quad C=A-1 \quad A=1/3$

$C=-2/3$

$$\int \frac{1/3}{x-1} + \frac{\frac{-x-2}{3}}{x^2+x+1}\,dx$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{3}\int \frac{x+2}{x^2+x+1}dx = \frac{1}{3}\ln|x-1| - \frac{1}{3}\int\frac{x+1/2\,dx}{x^2+x+1} - \frac{1}{3}\int\frac{3/2\,dx}{(x+1/2)^2+3/4}$$

let $u=x^2+x+1$

$du=2x+1$

$\frac{1}{2}du=x+1/2$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln|x+2| - \frac{1}{2}\left(\frac{2}{\sqrt{3}}\right)\tan^{-1}\left(\frac{x+1/2}{\sqrt{3}/2}\right)+C$$

31) $\int \frac{dx}{x(x^2+4)^2} = \frac{A}{x} + \frac{Bx+C}{(x^2+4)^1} + \frac{Dx+E}{(x^2+4)^2}$

$1 = A(x^2+4)^2 + [(Bx+C)(x)(x^2+4)] + [(Dx+E)(x)]$

$1 = Ax^4+8Ax^2+16A+Bx^4+Cx^3+4Bx^2+4Cx+Dx^2+Ex$

$x^4: 0=A+B \quad B=-1/16$

$x^3: 0=C$

$x^2: 0=8A+4B+D$

$x^1: 0=4C+E$

$x^0: 1=16A \quad A=1/16$

$A=1/16,\ B=-1/16,\ C=0,\ D=-1/4,\ E=0 \qquad 0=1/2-1/4+D \quad D=-1/4 \quad 0=4(0)+E \quad E=0$

$$=\int \frac{1/16}{x} + \frac{-1/16\,x}{(x^2+4)} + \frac{-1/4\,x}{(x^2+4)^2}dx = \frac{1}{16}\ln|x| - \frac{1}{16}\left(\frac{1}{2}\right)\ln|x^2+4| - \frac{1}{4}\left(\frac{-1}{2}\right)\frac{1}{x^2+4}+C$$

$$= \frac{1}{16}\ln|x| - \frac{1}{32}\ln|x^2+4| + \frac{1}{8(x^2+4}+C$$

33) $\int \frac{x-3}{(x^2+2x+4)^2}dx = \int \frac{x-3}{((x+1)^2+3)^2}dx$ Let $u=x+1$

$= \int \frac{u-4}{(u^2+3)^2}du = \int \frac{u\,du}{(u^2+3)^2} - 4\int\frac{1\,du}{(u^2+3)^2}$

$= \frac{u-4}{(u^2+3)^2} = \frac{Au+B}{(u^2+3)^1} + \frac{Cu+D}{(u^2+3)^2} \qquad = \int \frac{u-4}{(u^2+3)^2}$

$u-4 = (Au+B)(u^2+3)+Cu+D$

$u-4 = Au^3+3Au+Bu^2+3B+Cu+D$

$0=A \quad C=1 \quad B=0 \quad D=-4$

43)

$$\frac{-644}{323(1+2x)} - \frac{2358}{16031(-7+3x)} + \frac{23550}{4879(2+5x)} + \frac{4299+4190x}{52003(5+x+x^2)}$$

appendix G: 20, 30, 34

Skip LesteR 1/24/2013

20) $\int \frac{x^2-5x+16}{(2x+1)(x-2)^2}dx = \frac{A}{2x+1} + \frac{B}{(x-2)^1} + \frac{C}{(x-2)^2}$

$\vec{x}^2: 1 = A+2B$ $A=1-2B$

$x^1: -5=-4A-3B+2C$

$x^0: 16=4A-2B+C$

$x^2-5x+16 = A\underset{x^2-4x+4}{(x-2)^2} + B(2x+1)(x-2) + C(2x+1)$

$= Ax^2-4Ax+4A+(2Bx+B)(x-2)+2cx+C$

$x^2-5x+16 = Ax^2-4Ax+4A + 2Bx^2-4Bx+Bx-2B+2Cx+C$

$16=4(1-2B)-2B+C$

$16 = 4-8B-2B+C$

$12 = -10B+C$

$12+10B=C$

$12-10=C$

$C=2$

$B = C/10 - 6/5$

$-5=-4(1-2B)-3B+2(10B+12)$

$-5=-4+8B-3B+20B+24$

$-5=25B+20$

$-20 \quad -20$

$-25=25B \quad B=-1$

$A=1-2(-1)$

$1+2$

$A=3$

$= \int \frac{3dx}{2x+1} + \frac{-1\,dx}{(x-2)^1} + \frac{2\,dx}{(x-2)^2} \Rightarrow 3\int \frac{1dx}{2x+1} - \int \frac{1dx}{x-2} + \int \frac{2dX}{x^2-4x+4}$

let $u=2x+1$

$du=2dx$

$= \frac{3}{2}\int \frac{du}{u} - \int \frac{dx}{x-2} + 2\int \frac{1}{(x-2)^2}dx$ $(x-2)^{-2}$ $-(x-2)^{-1}$

$= \frac{3}{2}\ln|1+2x| - \ln|x-2| - (2/(x-2)) + C$

30) $\int \frac{x^3}{x^3+1}dx = \int \frac{x^3}{(x+1)(x^2-x+1)}$

$(x^3+1)\overline{)\,x^3+0\,}$ quotient 1; $-(x^3+1)$; -1 $= 1 - \frac{1}{x^3+1}$

$= \int 1dx - \int \frac{1}{x^3+1}dx \quad = \int 1dx - \left[\int \frac{1}{3(x+1)} + \left[\frac{(-x+2)}{3(x^2-x+1)}\right]dx\right]$

$x^2-x+1=0$

$\Rightarrow x^2-x=-1$

$\Rightarrow x^2-x+1/4=-3/4$

$\Rightarrow (x-1/2)^2=-3/4$

$= \int 1dx - \underbrace{\int \frac{1}{(x+1)(x^2-x+1)}dx}$ $= x - \left[\frac{1}{3}\int \frac{1}{x+1} - \frac{1}{3}\int \frac{x-2}{(x^2-x+1)}dx\right]$

$= \int \frac{1\;dx}{(x+1)(x^2-x+1)} = \int \frac{A}{(x+1)} + \frac{Bx+C}{(x^2-x+1)}dx$

$= x - \frac{1}{3}\ln(x+1) + 1/3\int\left(\frac{x-2}{x^2-x+1}\right)dx$

$= x - \frac{1}{3}\ln(x+1) + \frac{1}{3}\int\left(\frac{1}{2}\cdot\frac{2x-4}{x^2-x+1}\right)dx$

$1 = Ax^2-Ax+1A+(Bx+C)(x+1)$

$1 = Ax^2-Ax+1A+[Bx^2+Bx+Cx+C]$

$= x - \frac{1}{3}\ln(x+1) + \frac{1}{6}\left[\int \frac{2x-1}{x^2-x+1} - \frac{3}{x^2-x+1}dx\right]$

$1 = Ax^2-Ax+1A+Bx^2+Bx+Cx+C$

let $u = x^2-x+1$, $du = 2x-1\,dx$

$1=A+C$ $0=-A+B+C$ $0=A+B$

$C=1-A$ $-A=B$

$C=2/3$

$0=-A-A+1-A$ $B=-1/3$

$0=-3A+1$

$-1=-3A$ $A=1/3$

$= x - 1/3\ln(x+1) + 1/6\left[\int \frac{du}{u} - 3\int \frac{1}{x^2-x+1}\right]$

$= x - \frac{1}{3}\ln(x+1) + \frac{1}{6}\left[\ln(x^2-x+1) - 3\int \frac{1}{(x-\frac{1}{2})^2+3/4}\right]$

let $\omega=(x-\frac{1}{2})$

$= x - \frac{1}{3}\ln(x+1) + \frac{1}{6}\left[(\ln(x^2-x+1)) - 3\int \frac{1}{\omega^2+(\sqrt{3/4})^2}\right] + C$

$= x - 1/3\ln(x+1) + 1/6\left[\ln(x^2-x+1) - 3\cdot(2/\sqrt{3})\tan^{-1}\left(\frac{x-1/2}{\sqrt{3/4}}\right)\right] + C$

$= x - 1/3\ln(x+1) + 1/6\ln(x^2-x+1) - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x-1/2}{\sqrt{3}/2}\right) + C$

$\frac{1}{\sqrt{3/4}}$

$\frac{1}{\frac{\sqrt{3}}{2}}$

$\frac{2}{\sqrt{3}}$

$\frac{1}{2\sqrt{3/4}}$

$\frac{1}{2\frac{\sqrt{3}}{2}}$

Appendix G #34

Skip Lester Calc II 1/24/2013

34) $\int \frac{3x^2+x+4}{x^4+3x^2+2}dx = \int\frac{3x^2+x+4}{(x^2+1)(x^2+2)}dx \rightarrow \int\frac{3x^2+x+4}{(x^2+1)(x^2+2)} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+2)}$

$3x^2+x+4 = (Ax+B)(x^2+2) + (Cx+D)(x^2+1)$ $A=-C$ $B=3-D$

$3x^2+x+4 = Ax^3+2Ax+Bx^2+2B+Cx^3+1Cx+Dx^2+D$

$0=A+C$ $3=B+D$ $1=2A+C$ $4=2B+D$ $1=-2(C)+C$

$4-2B=D$ $1=-C$ $C=-1$

$A=1$ $3=B+4-2B$ $D=2$

$-1=-B$ $B=1$

$= \int\left[\frac{x+1}{(x^2+1)} + \frac{-x+2}{(x^2+2)}\right]dx = \int\frac{x\,dx}{(x^2+1)} + \int\frac{1\,dx}{x^2+1} + \int\frac{2\,dx}{x^2+2} - \int\frac{x\,dx}{x^2+2}$

let $u=x^2+1$
$du=2x\,dx$

$=2\int\frac{1}{x^2+2}dx$

let $w=x^2+2$
$dw=2x\,dx$

$=\frac{1}{2}\int\frac{1}{u}du + [\tan^{-1}x] - \frac{1}{2}\int\frac{1}{w}dw + \left[\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]$

$=\int\frac{1}{\frac{x^2}{2}+1}dx$ let $y=\frac{x}{\sqrt{2}}$ $dy=\frac{1}{\sqrt{2}}dx$

no need for Abs; $x^2+|c|>0$ for $\mathbb{R}$

$=\sqrt{2}\int\frac{1}{y^2+1^2}dy$ $dx=\sqrt{2}dy$

$=\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$

$=\frac{1}{2}\ln(x^2+1) - \frac{1}{2}\ln(x^2+2) + \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$ ...

... $+\tan^{-1}(x) + C$

Some Ideas:

- Trinomial in denominator?
  - Don't panic.

can you use completing-the-square?

- aim for $\frac{\text{something}}{x^2+a^2}$

Binomial numerator? $\int\frac{x+1}{x^2+1} = \int\frac{x}{x^2+1} + \int\frac{1}{x^2+1}$

- Split the integral.

- Factor denominators completely before partial fractions

Look for the derivative of the denominator in the numerator

manipulate constants!

Get to Cos'X or sin'x

Chapter 5.7
1-17 odd

Skip Lester Calculus II 1/26/13

1. $\int \sin^3 x \cos^2 x \, dx$

$= \int \sin^2 x \cos^2 x \sin x \, dx$

$= \int (1-\cos^2 x) \cos^2 x \sin x \, dx$

$\Rightarrow$ let $w = \cos x$ $dw = -\sin x \, dx$

$= \int (1-w^2) w^2 (-dw)$

$= \int (w^2 - 1) w^2 dw$

$= \int |w^4 - w^2| \, dw$

$= \frac{w^5}{5} - \frac{w^3}{3} + C$

$= \underline{\frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + C}$

5) $\int_0^{2\pi} \cos^2(6\theta) d\theta$ double Angle

$= \int_0^{2\pi} \frac{1}{2} [\cos(2 \cdot 6\theta) + 1) d\theta$

$= \frac{1}{2} \int_0^{2\pi} \cos 12\theta + 1 \, d\theta$

$= \frac{1}{2} \left[\frac{\sin 12\theta}{12} + \theta \right]_0^{2\pi}$

$= 1/2 [0 + 2\pi] - \frac{1}{2}[0+0]$

$= \underline{\pi}$

9) Use the substitution $u = \tan x$ to evaluate

$du = \sec^2 x dx$

$\int_0^{\pi/4} \tan^2 x \sec^4 x \, dx$

$= \int_0^{\pi/4} \tan^2 x (1 + \tan^2 x)(\sec^2 x dx)$

$= \int_0^1 u^2 (1 + u^2) du$

$= \int_0^1 u^2 + u^4 du$

$= \left[\frac{u^5}{5} + \frac{u^3}{3} \right] du$

$\frac{1}{5} + \frac{1}{3}$

$= \frac{3}{15} + \frac{5}{15}$

$= \underline{\frac{8}{15}}$

$\frac{A/H}{O/H}$

3. $\int_{\pi/2}^{3\pi/4} \sin^5 x \cos^3 x dx$

$= \int_{\pi/2}^{3\pi/4} \sin^5 x \cos x (1 - \sin^2 x) dx$

Let $u = \sin x$, $du = \cos x dx$

$= \int_1^{\sqrt{2}/2} u^5 (1 - u^2) du$

$= \int_1^{\sqrt{2}/2} u^5 - u^7 du$

$= \underline{\frac{-11}{384}}$

$= \left[u^6/6 - u^8/8 \right]_1^{\sqrt{2}/2}$

$\frac{\left(\frac{\sqrt{2}}{2}\right)^6}{6} - \frac{\left(\frac{\sqrt{2}}{2}\right)^8}{8} - \left[\frac{1^6}{6} - \frac{1^8}{8} \right]$

$= \frac{\left(\frac{2^3}{2^6}\right)}{6} - \frac{\left(\frac{2^4}{2^8}\right)}{8} - \frac{1}{6} + \frac{1}{8} = \frac{\left(\frac{1}{2^3}\right)}{6} - \frac{\left(\frac{1}{2^4}\right)}{8} - \frac{1}{6} + \frac{1}{8}$

$= \frac{1/8}{6} - \frac{1/16}{8} - \frac{1}{6} + \frac{1}{8} = \frac{1}{48} - \frac{1}{128} - \frac{1}{6} + \frac{1}{8} = \frac{-11}{384}$

7) Use the substitution $u = \sec x$ and evaluate:

$du = \sec x \tan x \, dx$

$\int \tan^3 x \sec x \, dx$

• $\sec^2\theta = \tan^2\theta + 1$ •
$\sec^2\theta - 1 = \tan^2\theta$
$u^2 - 1$

$= \int (\sec^2 x - 1)(\sec x \tan x \, dx)$

$= \int (u^2 - 1) du \Rightarrow \frac{u^3}{3} - u + C$

$= \frac{\sec^3 x}{3} - \sec x + C$

11) Use the substitution $x = 3\sin\theta$, $-\pi/2 \le \theta \le \pi/2$ and the identity $\cot^2\theta = \csc^2\theta - 1$ to evaluate

$dx = 3\cos\theta d\theta$

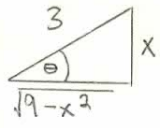
$\int \frac{\sqrt{9-x^2}}{x^2} dx = \int \frac{\sqrt{9 - 9\sin^2\theta}}{9\sin^2\theta} (3\cos\theta d\theta)$ $\Big\|$ $\sqrt{9 - 9\sin^2} = \sqrt{9\cos^2\theta}$

$= \int \frac{\sqrt{9\cos^2\theta}}{9\sin^2\theta} (3\cos\theta d\theta) = \int \frac{9\cos^2\theta}{9\sin^2\theta} d\theta = \int \frac{\cot^2\theta}{1} d\theta$ •identity time•

$= \int (\csc^2\theta - 1) d\theta \rightarrow -\cot\theta - \theta + C$

! $\sin\theta = \frac{x}{3}$

$\theta = \sin^{-1}\left(\frac{x}{3}\right)$



$\cot = \frac{\cos}{\sin}$

$= \frac{\sqrt{9-x^2}}{x}$

$= \boxed{\frac{-\sqrt{9-x^2}}{x} - \sin^{-1}\left(\frac{x}{3}\right) + C}$

Chapter 5.7 13-17

1/27/2013

Skip Lester Calculus II

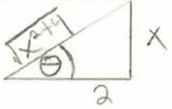
13) Use the substitution $x=2\tan\theta, -\frac{\pi}{2}<\theta<\frac{\pi}{2}$

to evaluate $dx = 2\sec^2\theta$

$$\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$$

$2\tan\theta = x$

$\tan\theta = x/2$



$\sin\theta = \frac{x}{\sqrt{x^2+4}}$

Trig Sub. Match dx to dθ

$$= \int \frac{2\sec^2\theta\,d\theta}{(2\tan\theta)^2\sqrt{(2\tan\theta)^2+4}}$$

$$= \int \frac{2\sec^2\theta}{4\tan^2\theta\sqrt{4\tan^2\theta+4}}\,d\theta$$

$\tan^2\theta + 1 = \sec^2\theta$

$$= \int \frac{2\sec^2\theta}{4\tan^2\theta(2\sqrt{\sec^2\theta})}\,d\theta$$

$$= \frac{1}{4}\int \frac{\sec^2\theta}{\tan^2\theta\sec\theta}\,d\theta$$

Trig rewrite

$$= \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

this sucks. break into sine and cosine so you can work WERQ.

$\sec\theta = \frac{1}{\cos\theta}$ $\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$

$$= \frac{1}{4}\int \frac{1}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{1}{4}\int \frac{\cos}{\sin^2\theta}\,d\theta$$

Let $u = \sin\theta$

$du = \cos\theta\,d\theta$

$$= \frac{1}{4}\int \frac{1\,du}{u^2}$$

$= \int u^{-2}du$

$= -u^{-1}+c$

$$= \frac{1}{4}\left[-\frac{1}{u}+C\right] = -\frac{1}{4\sin\theta}+C$$

$$= -\frac{1}{4}\cdot\frac{\sqrt{x^2+4}}{x}+C$$

$$\boxed{-\frac{\sqrt{x^2+4}}{4x}+C}$$

15) $$\int_{\sqrt{2}}^{2} \frac{1}{t^3\sqrt{t^2-1}}\,dt$$

Let $t = \sec\theta$ $\sec(\sqrt{2}) = \pi/4$

$dt = \sec\theta\tan\theta\,d\theta$ $\sec(2) = \pi/3$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1(\sec\theta\tan\theta\,d\theta)}{\sec^3\theta\sqrt{(\sec^2\theta-1)}}$$

Now, $\sec^2\theta - 1 = \tan^2\theta$

$\sqrt{\tan^2\theta} = \tan\theta$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cancel{\sec\theta\tan\theta}}{\sec^3\theta\,(\cancel{\tan\theta})}\,d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\sec^2\theta\,d\theta}$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos^2\theta\,d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left(\frac{\cos 2\theta + 1}{2}\right)d\theta$$

$\int \cos 2\theta\,d\theta$ let $u = 2\theta$

$du = 2d\theta$

$= \frac{1}{2}\int \cos u\,du$

$= \frac{1}{2}\sin u = \frac{\sin 2\theta}{2}+c$

$$= \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= \frac{1}{2}\left[\frac{\pi}{3}+\left(\frac{1}{2}\cdot\frac{\sqrt{3}}{2}\right)\right] - \frac{1}{2}\left[\frac{\pi}{4}+\left(\frac{1}{2}\cdot 1\right)\right]$$

$$= \frac{\pi}{6} + \frac{\sqrt{3}}{8} - \frac{\pi}{8} - \frac{2}{8}$$

$$= \frac{4\pi}{24} - \frac{3\pi}{24} + \frac{\sqrt{3}-2}{8}$$

$$\boxed{= \frac{\pi}{24} + \frac{\sqrt{3}-2}{8}}$$

5.7 17 4,6,12 13

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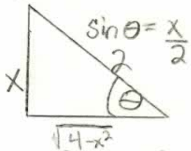
17) $\int \frac{dx}{x^2\sqrt{4-x^2}}$ Lets let $x=2\sin\theta$

$dx = 2\cos\theta d\theta$

$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

$= \int \frac{2\cos\theta d\theta}{4\sin^2\theta\sqrt{4-4\sin^2\theta}}$

$\sin\theta = \frac{x}{2}$



$= \int \frac{2\cos\theta d\theta}{4\sin^2\theta\sqrt{4\cos^2\theta}}$

$2^2 = x^2 + ?^2$

$4 - x^2 = ?^2$

$= \int \frac{\cancel{2\cos\theta} d\theta}{4\sin^2\theta(\cancel{2\cos\theta})}$ $4-4\sin^2\theta = 4\cos^2\theta$

$= \int \frac{d\theta}{4\sin^2\theta} = \frac{1}{4}\int \csc^2\theta d\theta$

$\frac{A}{O} = \cot$

$= \frac{-1}{4}\cot\theta + c$

$= \frac{-\sqrt{4-x^2}}{4x} + C$

4) $\int \sin^3(mx)dx$ Let $u = mx$

$du = mdx$

$= \int \sin^3(u)\frac{du}{m}$

$\frac{du}{m} = dx$

$= \frac{1}{m}\int (\sin^2(u)\sin(u)du$

$\cdot \sin^2x + \cos^2x = 1$

$= \frac{1}{m}\int \left(1-\cos^2(u)\right)\sin(u)du$

$\cdot$ Let $u = \cos u$

$du = -\sin u du$

$= \frac{-1}{m}\int (1-u^2)du = \frac{-1}{m}\left[u - \frac{u^3}{3}\right] + C$

$= \frac{-1}{m}\left(\cos u - \frac{\cos^3 u}{3}\right) + C$

$= \frac{-1}{m}\left(\cos(mx) - \frac{\cos^3(mx)}{3}\right) + C$

What's Wrong with #6) ?

6) $\int_0^{\pi/2} \sin^2 x\cos^2 x dx \Rightarrow \cdot \cos^2 x = \frac{1+\cos 2x}{2}$

$= \frac{1}{4}\int_0^{\frac{\pi}{2}} (1-\cos 2x)(1+\cos 2x)dx$ $\cdot \sin^2 x = \frac{1-\cos 2x}{2}$

$= \frac{1}{4}\int_0^{\pi/2} (1-\cos^2(2x)dx$

$\cdot (a-b)(a+b) = a^2-b^2$

let $w = 2x$

$dw = 2dx$

★re-do limits

$= \frac{1}{8}\int_0^{\frac{\pi}{4}} 1-\cos^2(w)dw$

$\frac{dw}{2} = dx$

$= \frac{1}{8}\int_0^{\pi/4} 1-\left[\frac{1+\cos 2w}{2}\right]dw$ $\cdot \cos^2 w = \frac{1+\cos 2w}{2} \cdot$

re-visit

$= \frac{1}{16}\int_0^{\pi/4} 2-1+\cos 2w dw$

$= \frac{1}{16}\int_0^{\pi/4} 1+\cos 2w dw$

$\frac{1}{16}\left[{}_0^{\frac{\pi}{4}} w + \frac{\sin 2w}{2}\right]_0^{\frac{\pi}{4}}$

$= \frac{w + \sin(w)(\cos(w))}{16}\Bigg|_0^{\pi/4}$

$= \frac{\pi}{4\cdot 16} + \frac{[\sqrt{2}/2][\sqrt{2}/2]}{16}$

$= \frac{\pi}{64} + \left(\frac{2}{4}\cdot\frac{1}{16}\right)$

$= \frac{\pi}{64} + \frac{1}{32}$ □

wrong↑

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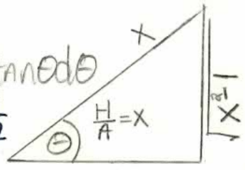
12) *Use the substitution*

$x = \sec\theta$ $dx = \sec\theta\tan\theta\, d\theta$

$0 \le \theta < \frac{\pi}{2}$

to evaluate



$$\int \frac{\sqrt{x^2-1}}{x^4}\,dx = \int \frac{\sqrt{\sec^2\theta - 1}}{\sec^4\theta} \cdot (\sec\theta\tan\theta\, d\theta)$$

$$\sec^2\theta - 1 = \tan^2\theta \rightarrow \sqrt{\tan^2\theta} = \tan\theta$$

$$= \int \frac{\tan\theta(\sec\theta\tan\theta)\,d\theta}{\sec^4\theta} = \int \frac{\tan^2\theta\, d\theta}{\sec^3\theta}$$

$$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta} \;\&\; \sec^3\theta = \frac{1}{\cos^3\theta}$$

$$= \int \frac{\sin^2\theta}{\cancel{\cos^2\theta}} \cdot \cos^3\theta\, d\theta$$

let $u = \sin\theta$

$du = \cos\theta\, d\theta$

$$= \int \sin^2\theta\cos\theta\, d\theta$$

$$= \int u^2\,du = \frac{u^3}{3} + C$$

$$= \frac{\sin^3\theta}{3} + C \qquad \sin\theta = \frac{\sqrt{x^2-1}}{x}$$

$$= \frac{1}{3}\left(\frac{\sqrt{x^2-1}}{x}\right)^3 + C$$

$$= \frac{(x^2-1)^{3/2}}{3x^3} + C$$

$x^2 = 1^2 + (\sqrt{x^2-1})^2$

$x = 1^2 + x - 1^2$

$x^2 = 1^2 + b^2$

$x^2 - 1 = b^2$

$\sqrt{x^2-1} = b$

$(1^2 + \sqrt{x^2-1})^2$

25) $$\int \frac{10}{(x-1)(x^2+9)}\,dx = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$$

$10 = Ax^2 + 9A + Bx + C(x-1)$

$10 = 9A - C$ $+ Bx^2 - Bx + Cx - C$

$10 - 9A = -C$ $0 = -B + C$ $0 = A + B$

$-10 + 9A = C$ $0 = A - 10 + 9A$ $-B = A$

$C = -1$ $10 = 10A$ $A = 1$ $B = -1$

$$= \int \frac{1}{x-1} + \underbrace{\frac{-x-1}{x^2+9}}_{} = \int \frac{-x}{x^2+9} - \int \frac{1}{x^2+9}$$

$$= \ln|x-1| - \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) - \frac{1}{2}\ln(x^2+9)$$

19) *Write in partial fraction expansion*

a) $$\frac{2x}{(x+3)(3x+1)} = \frac{A}{x+3} + \frac{B}{3x+1}$$

$2x = A(3x+1) + B(x+3)$

b) $$\frac{1}{x^3+2x^2+x} = \frac{1}{x(x^2+2x+1)}$$

$$= \frac{1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{(x+1)^1} + \frac{C}{(x+1)^2}$$

21) $$\int \frac{5x+1}{(2x+1)(x-1)}\,dx = \frac{A}{2x+1} + \frac{B}{x-1}$$

$5x + 1 = A(x-1) + B(2x+1)$

$5x + 1$ $Ax - 1A + 2Bx + B$

$5 = A + 2B$ $1 = -1A + B$

$B = 1 + 1A$

$5 = A + 2(1+A)$ $B = 2$

$5 = 3A + 2$

$-2 \quad -2$

$A = 1$

$$= \int \frac{1}{2x+1} + \frac{2}{x-1}\,dx$$

$$= \frac{1}{2}\int \frac{1\,dx}{x+\frac{1}{2}} + 2\int \frac{1\,dx}{x-1} = \frac{1}{2}\ln\left|x+\frac{1}{2}\right| + 2\ln|x-1| + C$$

23) $$\int_2^3 \frac{1}{x^2-1}\,dx = \int_2^3 \frac{1}{(x+1)(x-1)} = \frac{A}{(x+1)} + \frac{B}{(x-1)}$$

2SQUARES

$1 = Ax - A + Bx + B$

$1 = B - A$ $0 = A + B$

$1 + A = B$ $0 = A + 1 + A$

$B = 1/2$ $-1 = 2A$

$A = -1/2$

$$= \int \frac{-1/2}{x+1} + \frac{1/2}{x-1}$$

$$= \frac{1}{2}\int \frac{-1}{x+1} + \frac{1}{x-1}$$

$$= \frac{1}{2}\Big[-\ln|x+1| + \ln|x-1|\Big]_2^3$$

$$\frac{-\ln 4 + \ln 2}{2} - \frac{-\ln 3 + 0}{2}$$

$$= \frac{-\ln 4 + \ln 3 + \ln 2}{2}$$

Chapter 5.7 27-31 odd

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27) $$\frac{x^3+x^2+2x+1}{(x^2+1)(x^2+2)} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+2)}$$

$$x^3+x^2+2x+1 = [Ax+B(x^2+2)]+[Cx+D(x^2+1)]$$
$$= Ax^3+2Ax+Bx^2+2B+Cx^3+Cx+Dx^2+D$$

x^0 : $1=2B+D$ $D=1-2B$ $D=1$

x^1 : $2=2A+C$ $2=2A+1-A$ $1=A$

x^2 : $1=B+D$ $1=B+1-2B$ $0=-B$

x^3 : $1=A+C$ $C=1-A$ $C=0$

$$=\int \frac{x}{x^2+1} + \int \frac{1}{x^2+2} = \boxed{\frac{1}{2}\ln(x^2+1) + \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C}$$

29) Use long division to evaluate the integrals

$$\int \frac{x}{x-6}\,dx \qquad \begin{array}{r} 1 \\ x-6\,\overline{)\;x+0} \\ -(x-6) \\ \hline 6 \end{array}$$

$$\Rightarrow \int\left(1+\frac{6}{x-6}\right)dx = \int 1\,dx + \int \frac{6}{x-6}\,dx$$

$$= \int 1\,dx + 6\int \frac{1}{x-6}\,dx = \underline{\underline{x+6\ln|x-6|+C}}$$

31) $$\int \frac{x^3+4}{x^2+4}\,dx \qquad \begin{array}{r} x \\ x^2+4\,\overline{)\;x^3+0x^2+0x+4} \\ -(x^3+0x^2+4x+0) \\ \hline 0+0-4x+4 \end{array}$$

$$= \int x\,dx + \int \frac{-4x+4}{x^2+4}\,dx$$

$$= \frac{x^2}{2} - 4\int \frac{x}{x^2+4} + 4\int \frac{1}{x^2+4} + C$$

$$= \frac{x^2}{2} - 2\ln(x^2+4) + 2\tan^{-1}\left(\frac{x}{2}\right) + C$$

For
1-21, Use a table of Integrals
and evaluate

Chapter 5.8
1-29 odd

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① $\int \tan^3(\pi x)\,dx$ Let $u=\pi x$, $dx = \frac{du}{\pi}$

$= \frac{1}{\pi}\int \tan^3(u)\,du$ #69.

$= \frac{1}{\pi}\left[\frac{1}{2}\tan^2 u + \ln|\cos u| + C\right]$

$= \dfrac{\tan^2(\pi x) + 2\ln|\cos \pi x| + C}{\pi}$

⑤ $\int e^{2x}\arctan(e^x)\,dx$ let $u=e^x$, $du = e^x dx$, $dx = \frac{du}{u}$

$= \int u^2 \tan^{-1}(u)\left(\frac{du}{u}\right)$

#92.

$= \int u \tan^{-1}(u)\,du$

$= \left(\frac{u^2+1}{2}\right)\tan^{-1}(u) - \frac{u}{2} + C$

$= \frac{e^{2x}+1}{2}\tan^{-1}(e^x) - \frac{e^x}{2} + C$

⑨ $\int \frac{\tan^3(\frac{1}{z})}{z^2}\,dz$ Let $s = \frac{1}{z}$, $ds = \frac{-1}{z^2}dz$

#69.

$= \int \frac{-\tan^3(s)\,ds}{1} = -\int \tan^3(s)\,ds$

$= -\left[\frac{1}{2}\tan^2 s + \ln|\cos(s)| + c\right]$

$= -\frac{1}{2}\tan^2\left(\frac{1}{z}\right) - \ln\left|\cos\left(\frac{1}{z}\right)\right| + C$

③ $\int \frac{dx}{x^2\sqrt{4x^2+9}}$ let $u=2x$ $(a=3)$, $du = 2dx$

$\frac{1}{2} \div \frac{1}{4} = 2$

$= \int \frac{\frac{1}{2}du}{(\frac{u}{2})^2\sqrt{u^2+a^2}} = \int \frac{\frac{1}{2}du}{\frac{1}{4}u^2\sqrt{u^2+a^2}} = 2\int \frac{du}{u^2\sqrt{u^2+a^2}}$

#28. $= 2\left[\frac{-\sqrt{a^2+u^2}}{a^2u} + C\right] = \frac{-\cancel{2}\sqrt{9+(2x)^2}}{9(\cancel{2}x)} + C$

$= \frac{-\sqrt{9+4x^2}}{9x} + C$

⑦ $\int_0^\pi x^3\sin(x)\,dx$ #84. $\int u^n \sin(u)\,du = -u^n\cos(u) + n\int u^{n-1}\cos u\,du$

$= -x^3\cos x + 3\int x^2\cos x\,dx$

#85. $\int u^n \cos u\,du = u^n\sin(u) - n\int u^{n-1}\sin u\,du$

$= -x^3\cos x + 3\left[x^2\sin x - 2\int x^1 \sin x\,dx\right.$

$= -x^3\cos x + 3x^2\sin x - 6\left[-x\cos x + \underbrace{1\int (x^0)\cos x\,dx}_{\sin x + C}\right]$

$= -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + C\Big|_0^\pi$

$= -\pi^3\cos\pi + 3\pi^2\sin\pi + 6\pi\cos\pi - 6\sin\pi$

$= \pi^3 - 6\pi$

⑪ $\int y\sqrt{6+4y-4y^2}\,dy$ • Let $z = 6+4y-4y^2\,dy$ Keep equal

$z = 6-(4y^2-4y+1)+1$

$z = 7-(2y-1)^2$

$z = a^2-u^2$, $a=\sqrt{7}$, $u=(2y-1)$

$du = 2dy$ $2y = u+1$, $y = \frac{u+1}{2}$

$= \int y\sqrt{z}\,dy =$

$= \int \frac{u+1}{2}\sqrt{7-(2y-1)^2}\,\frac{du}{2}$

$= \frac{1}{4}\int (u+1)\sqrt{7-u^2}\,du$

#30.

$= \frac{1}{4}\int u\sqrt{a^2-u^2}\,du + \frac{1}{4}\int \sqrt{a^2-u^2}\,du$ $= \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$

$= \frac{u}{8}\sqrt{a^2-u^2} + \frac{a^2}{8}\sin^{-1}\left(\frac{u}{a}\right) + \frac{1}{4}\int u\sqrt{a^2-u^2}\,du$

Lets ~~let~~ $S = a^2-u^2$, $dS = -2u\,du$

$= -\frac{1}{8}\int \sqrt{S}\,dS$

$= \frac{-S^{3/2}}{12}$

$= \frac{-(6+4y+4y^2)^{(3/2)}}{12} + \left(\frac{2y-1}{8}\right)\sqrt{7-(2y-1)^2} + \frac{7}{8}\sin^{-1}\left(\frac{2y-1}{\sqrt{7}}\right) + C$

Chapter 5.8

13-29 odd

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⑬ $\int \sin^2 x \cos x \ln(\sin x)\,dx$

Let $u = \sin x$; $du = \cos x\,dx$

$= \int u^2 \ln(u)\,du$ #101.

$= \frac{u^{n+1}}{(n+1)^2}\left[(n+1)\ln u - 1\right] + C \quad n=2$

$= \frac{\sin^3 x}{9}\left[3\ln(\sin(x) - 1\right] + C$

⑮ $\int \frac{e^x\,dx}{3 - e^{2x}}$ Let $u = e^x$, $du = e^x dx$, Let $a = \sqrt{3}$

$= \int \frac{du}{a^2 - u^2}$ #19.

$= \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$

$= \frac{1}{2\sqrt{3}}\ln\left|\frac{e^x + \sqrt{3}}{e^x - \sqrt{3}}\right| + C$

⑰ $\int \frac{x^4\,dx}{\sqrt{x^{10} - 2}}$ Let $u = x^5$, $a = \sqrt{2}$, $du = 5x^4\,dx$

$= \frac{1}{5}\int \frac{du}{\sqrt{u^2 - a^2}}$ #43. $= \frac{1}{5}\ln\left|u + \sqrt{u^2 - a^2}\right| + C$

$= \frac{1}{5}\ln\left|x^5 + \sqrt{x^5 - 2}\right| + C$

⑲ $\int \frac{\sqrt{4 + (\ln(x))^2}}{x}\,dx$ Let $u = \ln x$, $du = \frac{1}{x}dx$

$= \int \sqrt{4 + u^2}\,du$ #21.

$= \frac{u}{2}\sqrt{4 + u^2} + \frac{2^2}{2}\ln\left(u + \sqrt{4 + u^2}\right) + C$

$= \frac{\ln x}{2}\sqrt{4 + (\ln x)^2} + 2\ln\left((\ln x) + \sqrt{4 + (\ln x)^2}\right) + C$

㉑ $\int \sqrt{e^{2x} - 1}\,dx$ Let $u = e^x$, $du = e^x dx$

$= \int \sqrt{u^2 - 1}\,\frac{du}{u}$ #41. $= \sqrt{u^2 - a^2} - a\cos^{-1}\frac{a}{|u|} + C$

$= \sqrt{e^{2x} - 1} - \cos^{-1}(e^{-x}) + C$

㉓ Verify by differentiation and by making the substitution $t = a + bu$ formula 53, given by $\int \frac{u^2\,du}{(a+bu)^2} = \frac{1}{b^3}\left(a + bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right)$

(a) $\frac{d}{du}\left[\frac{1}{b^3}\left(a + bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right)\right]$ $\quad \frac{d}{dx}\frac{a^2}{a+bx} = a^2 \cdot \frac{d}{dx}\left(\frac{1}{a+bx}\right)$ let $u = a+bx$ $\quad \frac{d}{du}\frac{1}{u} = -u^{-2}$

$\Rightarrow \frac{1}{b^3}\left[(b) + \frac{a^2 b}{(a+bu)^2} - \frac{2ab}{a+bu}\right] = \frac{1}{b^3}\left[\frac{\overset{b(a^2+2abu+b^2u^2)}{b(a+bu)^2} + a^2b - 2ab(a+bu)}{(a+bu)^2}\right] = \frac{1}{b^3}\left[\frac{b^3u^2 + \cancel{2ab^2u} + \cancel{a^2b} + \cancel{a^2b} - [\cancel{2a^2b} + \cancel{2ab^2u}]}{(a+bu)^2}\right]$

$= \frac{1}{\cancel{b^3}} \cdot \frac{\cancel{b^3}u^2}{(a+bu)^2} = \frac{u^2}{(a+bu)^2}$ $\quad$ (b) $dt = b\,du$

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25) $\int \sec^4 x\,dx$ // Mathematica gives $2/3 \tan x + 1/3 \tan x \sec^2 x$

$\frac{2}{3}\int \sec^2 x\,dx + \frac{1}{3} \tan x \sec^2 x$

27) $\int x^2\sqrt{x^2+4}\,dx$ Mathematica gives $1/4\, x(2+x^2)\sqrt{3+x^2} - 2\sin^{-1}(x/2)$

$= \frac{1}{4}\left[x(2+x^2)\sqrt{4+x^2} - 8\log\left(\frac{1}{2}\left(x+\sqrt{4+x^2}\right)\right)\right]$

29) $\int x\sqrt{1+2x}\,dx$ // Mathematica $\rightarrow \sqrt{1+2x}\left(\frac{2x^2}{5} + \frac{x}{15} - \frac{1}{15}\right)$

Chapter 5.9 3,5,11,15,17,19 Skip Lester January 30, 2013

C.A.S. 3) Estimate $\int_0^1 \cos(x^2)\,dx$ using (a) the Trapezoidal rule and (b) the midpoint rule, each with $n=4$. From a graph of the integrand, determine whether your answers are over-estimates or underestimates. What can you conclude about the true value of the integral? $\Delta x = (b-a)/n = (1-0)/4 = 1/4$

a) $T_4 = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)]$, $\Delta x = \frac{1}{4}$

$= \frac{1}{8}\left[f(0) + 2f(\tfrac{1}{4}) + 2f(\tfrac{1}{2}) + 2f(\tfrac{3}{4}) + f(1)\right]$

$= \frac{1}{8}\left[1 + 2\cos(\tfrac{1}{16}) + 2\cos\tfrac{1}{4} + 2\cos\tfrac{9}{16} + \cos 1\right] \approx .895759$

b) $M_4 = \Delta x[f(\bar{x}_1) + \ldots + f(\bar{x}_n)]$, $\bar{x} = \frac{x_{i-1} + x_i}{2} \rightarrow \bar{x}_i = \frac{1}{8}$, $\Delta x = \frac{1}{4}$

$= \frac{1}{4}\left[\cos\left(\tfrac{1}{8^2}\right) + \cos\left(\tfrac{3^2}{8^2}\right) + \cos\left(\tfrac{25}{64}\right) + \cos\left(\tfrac{7^2}{8^2}\right)\right] \approx .908907$

The graph of $\cos(x^2)$ is concave down on $[0,1]$, $\therefore$ T_4 is an underestimate, while M_4 is an overestimate. With these data, we infer the true value of

$T_4 < \int_0^1 \cos(x^2)\,dx < M_4$ $\quad .895759 < \int_0^1 \cos(x^2)\,dx < .908907$

5) Use (a) the midpoint rule and (b) simpson's rule to approximate the given integral with the specified value of n. Compare the approximations to the actual value to determine error. $\int_0^2 \frac{x}{1+x^2}\,dx$ $\quad n=10 \quad \frac{b-a}{n} \quad \frac{2}{10} = \frac{1}{5} = \Delta x$

(a) $\frac{1}{5}\left[f(\tfrac{1}{10}) + f(\tfrac{3}{10}) + f(\tfrac{5}{10}) + f(\tfrac{7}{10}) + f(\tfrac{9}{10}) + f(\tfrac{11}{10}) + f(\tfrac{13}{10}) + f(\tfrac{15}{10}) + f(\tfrac{17}{10}) + f(\tfrac{19}{10})\right] \approx \underset{M_{10}}{\underline{.806598}}$

(b) S_n $\frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$

$= \frac{1}{15}\left[f(0) + 4f(\tfrac{1}{5}) + 2f(\tfrac{2}{5}) \cdots + 2f(\tfrac{8}{5}) + 4f(\tfrac{9}{3}) + f(2)\right] \approx \underline{.804779}_{S_{10}}$

$\int_0^2 \frac{x\,dx}{1+x^2}$ Let $u = 1+x^2$, $du = 2x\,dx$ $= \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|1+x^2|\Big|_0^2 = \frac{1}{2}\ln 5 \approx \underline{.804719}_{\text{Actual}}$

Errors: Error M_{10} = Actual $- M_{10} = -0.001879$

Error S_{10} = Actual $- S_{10} \approx -.000060$

$T = 1+2+2\cdots 2+2+1$

Chapter 5.9

11, 15, 17, 19 $\quad S = 1+4+2+4+2+4+1$

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11) Approximate w/ T_8, M_8, & S_8

$$\int_0^{1/2} \sin(e^{t/2})dt \quad \Delta x = \frac{1/2 - 0}{8} = \frac{1}{16}$$

$$T_8 = \left(\frac{\Delta x}{2}\right)\left[f(x_0) + 2f(x_1) + \cdots 2f(x_{n-1}) + f(x_n)\right]$$

$$= \frac{1}{32}\left[f(0) + \left[2\sum_{i=1}^{7} f(x_i)\right] + f(x_8)\right] \approx \underline{.451948}$$

$$M_8 = \frac{1}{16}\left[f\left(\frac{1}{32}\right) + f\left(\frac{3}{32}\right)\cdots\right] \approx \underline{.451991}$$

$$S_8 = \frac{1}{48}\left[1f(0) + 4f\left(\frac{1}{16}\right) + 2f\left(\frac{2}{16}\right)\cdots 4f\left(\frac{7}{16}\right) + f\left(\frac{1}{2}\right)\right] \approx \underline{.451976}$$

15) $$\int_1^5 \frac{\cos x}{x} dx, \; n = 8 \quad \frac{b-a}{n} = \frac{5-1}{8} = \frac{1}{2} \quad x_i = \frac{2}{2} + i\left(\frac{1}{2}\right)$$

$$T_8 = \frac{1}{4}\left[f(0) + \left[2\sum_{i=1}^{7} f(x_i)\right] + \overset{\overset{f(5)}{\downarrow}}{f}(x_8)\right] = \underline{-0.495333}$$

$$M_8 = \frac{1}{2}\left[f\left(\frac{5}{4}\right) + f\left(\frac{7}{4}\right) + \cdots + f\left(\frac{19}{4}\right)\right] = \underline{-0.543321}$$

$$S_8 = \frac{1}{6}\left[1+4+2\cdots 2+4+1\right] \approx \underline{-.526123}$$

17) Find T_8 & M_8 for $\int_0^1 \cos(x^2)dx \quad \frac{b-a}{n} = \frac{1}{8} = \Delta x \quad x_i = 0 + i/8$ $\quad K >> \left|\max f''\right|_a^b$

$$T_8 = \frac{1}{16}\left[f(0) + 2\sum_{i=1}^{7} f(x_i) + f(1)\right] \approx \underline{.902333}$$

K a whole number

$$M_8 = \frac{1}{8}\left[f\left(\frac{1}{16}\right) + f\left(\frac{3}{16}\right) + \cdots + f\left(\frac{15}{16}\right)\right] \approx \underline{.905620}$$

K=4

Estimate Errors E_T & E_M $\quad E_T = \frac{K(b-a)^3}{12n^2} \quad E_M = \frac{K(b-a)^3}{24n^2}$

let $u = x^2$
$du = 2x dx$

$$E_T = \frac{4(1-0)^3}{12\cdot 8^2} = \frac{1}{192} \qquad E_m = \frac{4(1-0)^3}{24\cdot 8^2} = \frac{1}{384}$$

Chapter 5.9 *19, 21, 23, 25, 27, 29 Skip Lester February 1, 2013

19) Find the approximations T_{10}, M_{10}, and S_{10} for $\int_0^{\pi} \sin x \, dx$ and the corresponding errors E_T, E_m, E_S. $\Delta X = \frac{\pi - 0}{10} = \pi/10$

$$T_{10} = \frac{\Delta x}{2}\left[f(x_0) + 2\sum_{i=1}^{9} f(x_i) + f(x_{10})\right] \qquad E_T = \underline{.016476}$$

$$= \frac{\pi}{20}\left[\sin(0) + 2\sum_{i=1}^{9}\sin\left(\frac{i\pi}{10}\right) + \sin\pi\right] \approx \underline{1.983524}$$

$$M_{10} = \Delta x\left[\sum_{i=1}^{10} f\left(\frac{(2i-1)\pi}{20}\right)\right] \approx \underline{2.008248} \qquad \underline{E_m = .008248}$$

$$= \frac{\pi}{10}\left[\sum_{i=1}^{10} f\left(\frac{(2i-1)\pi}{20}\right)\right]$$

$$\int_0^{\pi} \sin x \, dx = -\cos x \Big|_0^{\pi} = 1 - (-1) = 2$$

$$S_{10} = \frac{\Delta x}{3}\left[f(x_0) + 4\sum_{i=1}^{5} f(x_{2i-1}) + 2\sum_{i=1}^{4} f(x_{2i}) + f(x_{10})\right] \qquad E_S = \underline{-0.000110}$$

$$= \frac{\pi}{30}\left[\sin(0) + 4\sum_{i=1}^{5}\sin\left(\frac{(2i-1)\pi}{10}\right) + 2\sum_{i=1}^{4}\sin\left(\frac{2i\pi}{10}\right) + \sin(\pi)\right] \approx \underline{2.000110}$$

Compare the actual errors to the error estimates given by

$$\underbrace{|E_T| \le \frac{K(b-a)^3}{12n^2}, \ |E_M| \le \frac{K(b-a)^3}{24n^2}}_{|f''(x)| \le K \text{ for } a \le x \le b}, \text{ and } \underbrace{|E_S| \le \frac{K(b-a)^5}{180n^4}}_{|f^{(4)}(x)| \le K}$$

$f''(x) = -\sin(x) \rightarrow$ for $f(x) = \sin x, \cos x, -\cos x, -\sin x$, $f^{(n)}(x) \le 1$ ∴ take $K = 1$

$E_T = \frac{\pi^3}{12(10^2)} \approx .0258386$ $\quad E_M^{\text{EXPECTED}} \ \frac{\pi^3}{24(10^2)} \approx .0129193$ $\quad E_S^{\text{exp.}} = \frac{\pi^5}{180(10^4)} = .000170011$

$E_T^{(\text{ACTUAL})} = .016476$

$E_m^{\text{ACTUAL}} = .008248$ $\quad E_S^{\text{ACT}} = .000110$

$E_T^{(E)}/E_T^{(A)} = .637 \rightarrow 63.7\%$

$E_M^{(e)}/E_M^{(A)} = .638 \rightarrow 63.8\%$ $\quad E_S^{\text{exp}}/E_S^{\text{ACT}} = .647 \rightarrow 64.7\%$

How large do we have to choose n so the T_n, M_n, + S_N estimates are accurate to 0.00001?

$T_n \rightarrow \frac{\pi^3}{12(n^2)} \le \frac{1}{10^5}$

$n^2 \ge \frac{10^5\pi^3}{12}$

$n \approx 508.3$, take $\underline{n = 509}$ for T_n

$M_n \rightarrow \frac{\pi^3}{24n^2} \le \frac{1}{10^5}$

$= \frac{10^5\pi^3}{24} \le n^2$

$359.4 \le n$

Take $\underline{360 = n}$ for M_n

$S_N \rightarrow \frac{\pi^5}{180n^4} \le \frac{1}{10^5}$

$\frac{10^5\pi^5}{180} \le n^4$

$20.3 \le n$

Take $\underline{n = 22}$ (next higest even whole #) for S_n

$I = \int_0^{2\pi} e^{\cos x}\,dx$

C.A.S + Mathematica Problems

Chapter 5.9 21-29 odd

Skip Lester Calculus II 2/2/2013

21) $F(x) = e^{\cos x}$

a) $f''(x) = e^{\cos x}(\sin^2 x - \cos x)$
whose |maximum| is on the
end points at $0, 2\pi = -e$
Take $k = e \underset{or}{\approx} 3$

c) $\frac{e(2\pi)^3}{24(10^2)} \approx 0.280945995$

d) $7.95492\,65\underline{210}\,12845\ldots$

f) $F^{(4)}(x) = e^{\cos x}(\sin^4 x - 6\sin^2 x\cos x + 3 - 7\sin^2 x + \cos x)$

$|f^{(4)}(x)|$ max $\Leftrightarrow x = 0, x = 2\pi$
$y = 4e$
Take $k = 4e$

b) $M_{10} \approx 7.9549265\underline{18}$

e) 3×10^{-9} off from exact. Wow!

g) $S_{10} \approx 7.953789422$

h) $\frac{4e(2\pi)^5}{180(10^4)} \approx 0.059153618$

i) .00114 actual error

j) $\frac{4e(2\pi)^5}{180n^4} \leq \frac{1}{10^4} \rightarrow \frac{4e\cdot 10^4\cdot(2\pi)^5}{180} \leq n^4 \quad n \geq \left(\frac{4e(10^4)(2\pi)^5}{180}\right)^{\frac{1}{4}} \approx 49.3 \rightarrow$ Take $n = 50$ (even whole)

23) Find $L_n, R_n, T_n,$ and M_n for $n = 5, 10, 20$ for $\int_0^1 xe^x\,dx$.
Compute the errors $E_{L_n}, E_{R_n}, E_{T_n}, E_{M_n}$

I observe that when n doubles, $E_L + E_R$ decrease by a factor of 2, while $E_T + E_M$ decrease by a factor of four.

$E_M < E_T <<< E_L = E_R$

25) Find the approximations $T_n, M_n,$ & S_n for $n = 6$ + $n = 12$, then compute errors E_T, E_M, E_S. What did you observe? $\int_0^2 x^4\,dx$

$E_T + E_M$ are always opposite in sign and decrease by a factor of <u>four</u> when n is doubled, while E_S decreased by a factor of <u>16.</u> $E_S <<< E_M$

27) $\Delta x = \frac{b-a}{n} = \frac{6-0}{6} = 1$

$T_6 = \frac{1}{2}[3 + 2(5) + 2(4) + 2(2) + 2(2.8) + 2(4) + 1] \approx 19.8$

$M_6 = [4.5 + 4.7 + 2.6 + 2.2 + 3.4 + 3.2] \approx 20.6$

$S_6 = 1/3[3 + (4.5) + (2\cdot4) + (4\cdot2) + (2\cdot2.8) + (4\cdot4) + 1] \approx 20.5\overline{3}$

(5.9)

29) By Net Change Theorem, $\Delta V = \int_0^6 a(t)\,dt \quad \underset{n=6\rightarrow}{} \frac{6-0}{6} = \Delta t = 1$

$\frac{1}{3}\left[0 + (4\cdot\frac{1}{2}) + (2\cdot4.1) + 4(9.8) + 2(12.9) + 4(9.5) + 0\right] = 37.7\overline{3}$ ft/s

Ch. 5.10 Improper Integrals
1, 5-35 odd, 41-45 odd

Skip Lester February 3, 2013

#1) Explain why each of the following integrals is improper

a) $\int_1^{\infty} x^4 e^{-x^4} dx$ Type I

b) $\int_0^{\frac{\pi}{2}} \sec x\, dx$ Type II

c) $\int_0^2 \frac{x}{x^2-5x+6} dx$ Type II

d) $\int_{-\infty}^0 \frac{1}{x+5} dx$ Type I

a and d have infinity in the limits of integration.

c has a vertical asymptote at $x=2$, while b has one at $x=\pi/2$

7) $\int_{-\infty}^{-1} \frac{1}{\sqrt{2-w}} dw \rightarrow \lim_{t\to-\infty} \int_t^{-1} \frac{1}{\sqrt{2-\omega}} d\omega$

$= \lim_{t\to-\infty} \int_t^{-1} (2-\omega)^{(-1/2)} d\omega \rightarrow \lim_{t\to-\infty} -2\sqrt{2-\omega}\Big|_t^{-1}$

$= \lim_{t\to-\infty} \left(-2\sqrt{3} + 2\sqrt{2-t}\right) = \underline{\infty}$ $\underline{\text{divergent}}$

13) $\int_{-\infty}^{\infty} xe^{-x^2} dx = \int_{-\infty}^{0} xe^{-x^2} dx + \int_0^{\infty} xe^{-x^2} dx$

$= \lim_{t\to-\infty} \int_t^0 xe^{-x^2} = \lim_{t\to-\infty} \left[\frac{-e^{-x^2}}{2}\right]_t^0$ $\lim_{t\to-\infty} \frac{-e^{-t^2}}{2} = 0$

$= \frac{-e^{0^2}}{2} - 0 = \frac{-1}{2}$

$= \lim_{z\to\infty} \int_0^z xe^{-x^2} dx$

$= \lim_{z\to\infty} \left[\frac{-e^{-x^2}}{2}\right]_0^z = 0 + \frac{1}{2}$

$\therefore$ the integral is $-\frac{1}{2} + \frac{1}{2} = \underline{0}$ $\underline{\text{Convergent}}$

Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

5) $\int_3^{\infty} \frac{dx}{(x-2)^{3/2}} \rightarrow \lim_{t\to\infty} \int_3^t \frac{dx}{(x-2)^{3/2}}$

$= \lim_{t\to\infty} \int_3^t (x-2)^{(-3/2)} dx = \lim_{t\to\infty} \left[-2(x-2)^{(-1/2)}\right]_3^t$

$= \lim_{t\to\infty} \frac{-2}{\sqrt{x-2}}\Big|_3^t = \lim_{t\to\infty}\left[\left(\frac{-2}{\sqrt{t-2}}\right) + \frac{2}{\sqrt{1}}\right] = 0+2 = \underline{2}$

$\underline{\text{the limit exists} \therefore \text{convergent}}$:)

9) $\int_4^{\infty} e^{-y/2} dy \rightarrow \lim_{c\to\infty} \int_4^c e^{-y/2} dy$

$= \lim_{c\to\infty} \left[-2e^{-y/2}\right]_4^c = \lim_{c\to\infty} -2e^{-c/2} - -2e^{-2}$

$= 0 + 2e^{-2}$ convergent $\underline{2e^{-2}}$

11) $\int_{2\pi}^{\infty} \sin\theta\, d\theta \rightarrow \lim_{x\to\infty} \int_{2\pi}^{x} \sin\theta\, d\theta$

$= \lim_{x\to\infty} \left[-\cos(x) + \cos(2\pi)\right]$ limit does not exist; $\underline{\text{divergent!}}$

15) $\int_1^{\infty} \frac{x+1}{x^2+2x} dx = \lim_{s\to\infty} \int_1^s \frac{x+1}{x^2+2x} dx$

$\Rightarrow \frac{x+1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$ | $x+1 = Ax+2A+Bx$
$1 = A+B$ $1 = 2A$ $A = 1/2 = B$

$= \lim_{s\to\infty} \int_1^s \frac{1}{2x} dx + \lim_{s\to\infty} \int_1^s \frac{1}{2x+4} dx$

$= \lim_{s\to\infty} \left[\frac{\ln x}{2}\right]_1^s + \lim_{s\to\infty} \left[\frac{\ln(2x+4)}{2}\right]_1^s = \infty$ $\underline{\text{divergent}}$

very slowly approaching infinity w/o bound. □

5.10 17-35 odd 41-45 odd

Skip LesteR
Calculus II February 4, 2013

17) $\int_0^{\infty} se^{-5s}\,ds \rightarrow \lim_{c\to\infty}\int_0^c se^{-5s}\,ds$ let $u=s$ $dv=e^{-5s}ds$, $du=ds$ $v=-\frac{1}{5}e^{-5s}$

$\int u\,dv = u\cdot v-\int v\,du$ $s\cdot -\frac{1}{5}e^{-5s}$ $\int -\frac{e^{-5s}}{5}ds$

$= \lim_{c\to\infty}\left[\frac{-e^{-5s}}{5}s-\frac{e^{-5s}}{25}\right]_0^c \rightarrow = \lim_{c\to\infty}\frac{-ce^{-5c}}{5}-\frac{e^{-5c}}{25} = \lim_{c\to\infty}\frac{-c}{5e^{5c}} \overset{Ⓗ}{\rightarrow} \frac{-1}{25e^{5c}} \lim c\to\infty = 0$

l'Hôpital

$= 0-0+\frac{1}{25}$ convergent

19) $\int_1^{\infty}\frac{\ln x}{x}dx \rightarrow \lim_{a\to\infty}\int_1^a \frac{\ln x\,dx}{x}$

let $u=\ln x$ $du=1/x\,dx$

$= \int_1^a u\,du = \frac{u^2}{2}\Big|_1^a \rightarrow \lim_{a\to\infty}\frac{\ln x^2}{2}\Big|_1^a$

$=$ ∞ divergent

21) $\int_{-\infty}^{\infty}\frac{x^2}{9+x^6}dx = \int_{-\infty}^{0}\frac{x^2}{9+x^6}dx + \int_0^{\infty}\frac{x^2}{9+x^6}dx$

$= \lim_{b\to\infty}\int_b^0\frac{x^2}{9+x^6}dx + \lim_{a\to\infty}\int_0^a\frac{x^2\,dx}{9+x^6}$

$= 2\int_0^{\infty}\frac{x^2}{9+x^6}$ (even func. integrand) $\rightarrow \lim_{a\to\infty}2\int_0^a\frac{x^2dx}{9+x^6}$

Let $u=x^3$ $du=3x^2dx$ $= \lim_{a\to\infty}2\int_0^a\frac{\frac{1}{3}du}{3^2-u^2}$

Let $u=3v$ $du=3dv \rightarrow$

$\lim_{a\to\infty}2\int_0^a\frac{1/3(3dv)}{3^2-3v^2} = \lim_{a\to\infty}\frac{2}{9}\int_0^a\frac{dv}{1+v^2}$

$= 2/9\left[\tan^{-1}v\right]_0^a = \frac{2}{9}\left[\tan^{-1}\frac{u}{3}\right]_0^a = \frac{2}{9}\left[\tan^{-1}\frac{x^3}{3}\right]_0^a$

$= 2\lim_{a\to\infty}\frac{1}{9}\tan^{-1}\frac{a^3}{3} = \frac{2}{9}\cdot\frac{\pi}{2} = \frac{\pi}{9}$ convergent

43) $\int_0^{\infty}\frac{x}{x^3+1}dx$

27) $\int_{-2}^{14}\frac{dx}{\sqrt[4]{x+2}}$

Let $u=x+2$, $du=dx$

$= \lim_{c\to -2^+}\int_c^{14}\frac{dx}{(x+2)^{(1/4)}} = \lim_{c\to -2^+}\int_{\square}^{\Delta}\frac{du}{u^{(1/4)}}$

$= \lim_{c\to -2^+}\left[\frac{4}{3}u^{(3/4)}\right] \Rightarrow \lim_{c\to -2^+}\left[\frac{4}{3}(x+2)^{(3/4)}\right]_c^{14}$

$= \frac{4}{3}\left(16^{3/4}\right) - \lim_{c\to -2^+}\left[\frac{4}{3}(c+2)^{3/4}\right]$

$= 32/3$

33) $\int_0^2 z^2\ln(z)\,dz$ let $u=\ln z$ $dv=z^2dz$, $du=1/z\,dz$ $v=\frac{1}{3}z^3$

$\int u\,dv = uv-\int v\,du$

$= \lim_{c\to 0^+}\left[\frac{z^3\ln z}{3}-\int\frac{z^2dz}{3}\right]_c^2$

$= \lim_{c\to 0^+}\left[\frac{z^3\ln z}{3}-\frac{1}{3}\left[\frac{z^3}{3}\right]_c^2\right]$

$= \left[\frac{8\ln 2}{3}-\frac{8}{9}\right]-\lim_{c\to 0^+}\left[\frac{c^3\ln c}{3}-\frac{c^3}{9}\right]$

5.10 47, 49?, 51, 55, 59, 33

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47) $\int_0^1 \frac{\sec^2 x}{x\sqrt{x}}dx = \int_0^1 \frac{1\,dx}{x\sqrt{x}\cos^2 x}$

comparing →

$\Rightarrow \frac{1\,dx}{x^{3/2}\cos^2 x} > \frac{1\,dx}{x^{(3/2)}}$

$\uparrow \quad \uparrow$

$0 \leq \cos^2 x \leq 1 \therefore$

$\int_0^1 x^{(-3/2)}dx = \frac{-2}{3}x^{-1/2}\Big|_0^1$

$= \frac{-2}{3\sqrt{1}} - \lim_{C\to 0^+} \frac{-2}{3\sqrt{C}}$

$= \frac{-2}{3} + \infty$ divergent

∴, by comparison,

$\int_0^1 \frac{\sec^2 x}{x\sqrt{x}}dx$ is divergent

49) $\int_0^\infty \frac{1\,dx}{\sqrt{x}\,(1+x)}$

51) Find the values of p for which the integral converges and evaluate the integral for those values of p.

$\int_0^1 \frac{1}{x^p}dx \quad p=1: \int_0^1 \frac{dx}{x} \to \lim_{C\to 0^+}\int_C^1 x^{-1}dx$

$= 1 - \infty$. divergent

Sneaky!

$p \neq 1$: $(p>0)$: $\int_0^1 \frac{1}{x^p}dx = \lim_{C\to 0^+}\left[\frac{x^{-p+1}}{-p+1}\right]_C^1$

$= \frac{1^{-p+1}}{-p+1} - \lim_{C\to 0^+}\left[\frac{C^{-p+1}}{-p+1}\right] = \lim_{C\to 0^+} \frac{1^{(-p+1)}}{-p+1} - \frac{1}{(-p+1)C^{(p-1)}}$

$\lim_{C\to 0^+}\left[\frac{1}{1-p}\left(1 - \frac{1}{C^{p-1}}\right)\right]$ || If $p>1$, $p-1>0$, and the integral diverges

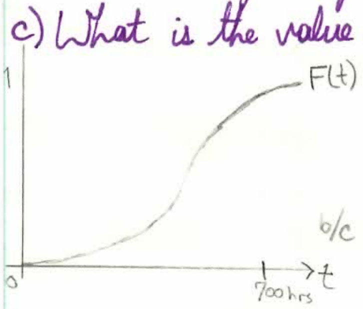
If $p<1$, $p-1<0$, and the integral converges

55) A manufacturer of lightbulbs wants to produce lightbulbs that last about 700 hours. However, some burn out faster than others. Let F(t) be a fraction of the company's bulbs that burnout before t hours, such that F(t) always evaluates between zero and one.

a) Sketch a guess graph b) what is the meaning of the derivative $r(t)=F'(t)$?

c) What is the value of $\int_0^\infty r(t)\,dt$? Why?

the rate at which the % of burnt out bulbs is increasing as time increased.



c) $\int_0^\infty r(t) = F(t)\Big|_0^\infty$

and $\lim_{t\to\infty} F(t) = 1$

b/c all lightbulbs burnout.

59) Determine how large the number a has to be so that

$\int_a^\infty \frac{1}{x^2+1}dx < 0.001$

$= \lim_{z\to\infty} \int_a^z \frac{1}{x^2+1}dx$

$\lim_{z\to\infty}\left[\tan^{-1}x\right]_a^z = \frac{\pi}{2} - \tan^{-1}a < .001$

$\tan^{-1}a > \pi/2 - .001$

"complex ∞" $\quad a = \tan(\pi/2 - .001)$

$a = 1000$

Chapter 5.10
23-35 odd, 41-45 odd

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23) $\int_e^{\infty} \frac{1}{x.(\ln x)^3} dx \to \lim_{c\to\infty} \int_e^c \frac{1}{x(\ln x)^3} dx$

let $u = \ln x$
$du = 1/x \, dx$ $\Rightarrow \int_1^{\ln c} u^{-3} du = -\frac{1}{2} u^{-2} \Big]_1^{\ln c}$

$= \lim_{c\to\infty} \frac{-1}{2u^2} \Big]_1^{\ln(c)} = \left[0 + \frac{1}{2} \right] = \underline{\frac{1}{2}}$ convergent.

25) $\int_0^1 \frac{3}{x^5} dx$ let $c \to 0^+ \Rightarrow \lim_{c\to 0^+} 3 \int_c^1 x^{-5}$

$= \lim_{c\to 0^+} 3 \left[\frac{-1}{4x^4} \right]_c^1 = \frac{-3}{4} + \infty$ divergent

27) on previous page

29) $\int_0^{33} (x-1)^{-1/5} dx = \int_0^{33} \frac{1}{\sqrt[5]{x-1}} dx$

the integrand is undefined for $x=1$

$= \int_0^1 \frac{1}{\sqrt[5]{x-1}} dx + \int_1^{33} \frac{1}{\sqrt[5]{x-1}} dx$ $\quad \int (x-1)^{-1/5} \searrow +\frac{5}{4}(x-1)^{4/5}$

$= \lim_{s\to 1^-} \int_0^s \frac{1}{\sqrt[5]{x-1}} dx + \lim_{s\to 1^+} \int_s^{33} \frac{1}{\sqrt[5]{x-1}} dx$

$= \lim_{s\to 1^-} \left[\frac{5}{4}(x-1)^{(4/5)} \right]_0^s + \lim_{s\to 1^+} \left[\frac{5}{4}(x-1)^{(4/5)} \right]_s^{33}$

$= 0 - \frac{5}{4} + [20 - 0] = \underline{\frac{75}{4}}$ convergent

31) $\int_{-1}^1 \frac{e^x \, dx}{e^x - 1}$ infinite discontinuity at $x=0$ $(e^0-1)=0$

$= \lim_{c\to 0^-} \int_{-1}^c \frac{e^x \, dx}{e^x-1} + \lim_{d\to 0^+} \int_d^1 \frac{e^x}{e^x-1}$

let $u = e^x - 1$
$du = e^x dx$ $\int \frac{du}{u} \to [\ln u + c]$

$\Rightarrow \ln(1-e^x) \to$

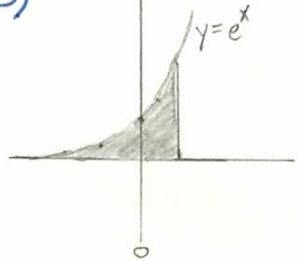
$\lim_{c\to 0^-} \left[\ln|e^x - 1| \right]_{-1}^c + \lim_{d\to 0^+} \left[\ln|e^x-1| \right]_d^1$

$\lim_{c\to 0^-} \ln(e^x - 1) = -\infty \to$ divergent.

33) cont previous page

Another Day. !!
$= -\lim_{c\to 0^+} \left[\frac{c^3 \ln c}{3} - \frac{c^3}{9} \right]$
l'hopital

35) Sketch $\Rightarrow S = \{\{x,y\} \mid x \le 1, 0 \le y \le e^x\}$



41) C.A.S. ✓ $g(x) = \frac{\sin^2 x}{x^2}$

b) $\sin^2 x \in [0,1]$

$\therefore \frac{\sin^2 x}{x^2} \le \frac{1}{x^2} \therefore$

since $\frac{1}{x^2}$ is convergent,

so too is $\frac{\sin^2 x}{x^2}$

43) compare:
$\int_0^{\infty} \frac{x}{x^3+1} < \frac{x}{x^3} = \frac{1}{x^2}$

therefor convergent.

45) $\int_1^{\infty} \frac{x+1}{\sqrt{x^4 - x}} \approx \frac{x}{\sqrt{x^4}} = \frac{x}{x^2} = \frac{1}{x} > \frac{1}{x^2}$

$\therefore$ this integral is divergent.

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33) $$\int_0^2 Z^2 \ln z\,dZ = \lim_{c\to 0^+}\int_c^2 Z^2 \ln z\,dz$$

Let $u = \ln z$ $dv = z^2 dv$

$du = \frac{1}{z}dz$ $v = \frac{z^3}{3}$ $\int z^2 = \searrow \frac{z^3}{3}+c$

$$= \lim_{c\to 0^+}\left[\frac{z^3\ln z}{3} - \int_c^2 \frac{z^3\,dz}{3z}\right]_c^2$$

$$= \lim_{c\to 0^+}\left[\frac{z^3\ln z}{3} - \frac{1}{3}\left[\frac{z^3}{3}\right]\right]_c^2$$

$$= \lim_{c\to 0^+}\left[\frac{1}{3}\left(z^3\ln z - \frac{z^3}{3}\right)\right]_c^2$$

$$= \frac{8\ln 2}{3} - \frac{8}{9} - \left[\lim_{c\to 0^+}\left[\frac{1}{3}\left(\frac{c^3\ln c}{1} - \frac{c^3}{3}\right)\right]\right]$$

$$= \frac{8\ln 2 - \sqrt{8}}{3} - \left[\lim_{c\to 0^+}\frac{\ln c}{3c^{-3}} - \frac{c^3}{9}\right]$$

← goes to 0

$$= \lim_{c\to 0^+}\frac{1}{3}\,\frac{\ln c}{c^{-3}} \overset{\boxed{H'}}{=} \quad \frac{1}{3}\cdot\frac{1}{c}\cdot\frac{1}{c^{-4}}\cdot\frac{-1}{3}$$

$$= \lim_{c\to 0^+}\left[\frac{1}{9}\cdot\frac{c^4}{c}\right] = \lim_{c\to 0^+}\frac{c^3}{9} = 0$$

→ $\frac{8\ln 2 - \sqrt{8}}{3}$ convergent

5.Review

31) $$\int e^{\sqrt[3]{x}}\,dx$$

let $u = x^{(1/3)}$ $x = u^3$

$du = \frac{x^{(-2/3)}\,dx}{3}$ $dx = 3x^{2/3}du$

$dx = 3u^2 du$

$$= \int e^u (3u^2)\,du$$

$$= \int 3u^2 e^u\,du$$

$$= 3\int u^2 e^u\,du = 3[u^2e^u + 2ue^u + 2e^u] + C$$

$$= 3\left(x^{2/3}e^{x^{(1/3)}} + 2x^{(1/3)}e^{x^{(1/3)}} + 2e^{x^{(1/3)}}\right) + C$$

$$\begin{array}{ccc} u^2 & \xrightarrow{+} & e^u \\ 2u & \xrightarrow{-} & e^u \\ 2 & \xrightarrow{+} & e^u \\ 0 & & e^u \end{array}$$

5.10 #49) $$\int_0^\infty \frac{1}{\sqrt{x}(1+x)}\,dx$$

let $u = \sqrt{x}$ $x = u^2$

$dx = 2u\,du$

$$= \int_0^1 \frac{1\,dx}{\sqrt{x}(1+x)} + \int_1^\infty \frac{1\,dx}{\sqrt{x}(1+x)}$$

$$= \lim_{c\to 0^+}\int_c^1 \frac{1\,dx}{\sqrt{x}(1+x)} + \lim_{d\to\infty}\int_1^d \frac{1\,dx}{\sqrt{x}(1+x)}$$

$$= \lim_{c\to 0^+}\left[2\tan^{-1}\sqrt{x}\right]_c^1 + \lim_{d\to\infty}\left[2\tan^{-1}\sqrt{x}\right]_1^d$$

$$\pi/2 - 0 + \pi - \pi/2 = \underline{\pi}$$

$$= \int \frac{2u\,du}{u(1+u^2)}$$

$$= 2\int \frac{u}{u(1+u^2)}\,du$$

$$= 2\int \frac{1}{1+u^2}\,du$$

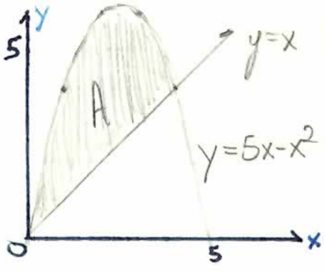
$$= 2\tan^{-1}u + C$$

$$= 2\tan^{-1}\sqrt{x} + C$$

Skip Lester February 7, 2013

Chapter 6.1

#1)



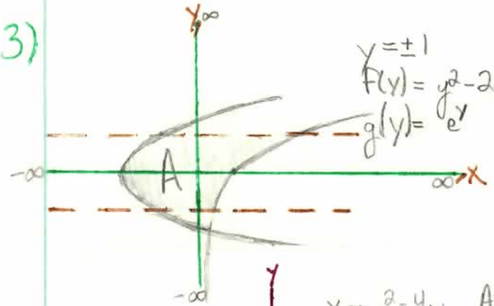
Find the area of the shaded region

$= \int [5x-x^2]dx - \int xdx$

$= \frac{5}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{2}x^2 \Big]_0^4 = \frac{5}{2}(16) - \frac{1}{3}(64) - \frac{1}{2}(16) - 0$

$= 40 - 64/3 - 8 = 32 - 64/3 = \underline{32/3}$

3)



$y = \pm 1$

$f(y) = y^2-2$

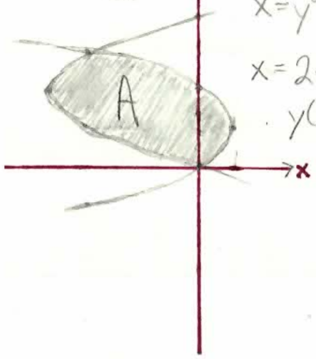
$g(y) = e^y$

$A = \int_{-1}^{1} e^y - (y^2-2)dy$

$= e^y - \frac{y^3}{3} + 2y \Big]_{-1}^{1} = e - 1/3 + 2 - [1/e + 1/3 - 2]$

$= \underline{e - 1/e + 10/3}$

4)



$x = y^2 - 4y$

$x = 2y - y^2$

$y(2-y)$

$A = \int_0^3 2y - y^2 - (y^2-4y)\, dy$

$A = \int 6y - 2y^2 dy$

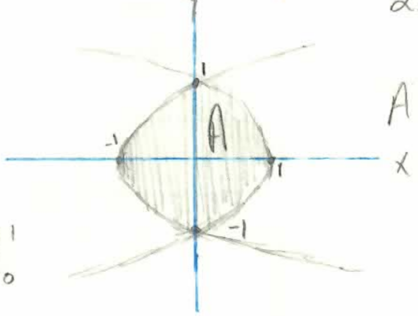
$= 3y^2 - \frac{2}{3}y^3 \Big]_0^3 = 3 \cdot 3^2 - [(2/3) \cdot 27] - 0 - 0$

$27 - 18 = \underline{9}$

9) $x = 1 - y^2,\ x = y^2 - 1 \Rightarrow 1 - y^2 = y^2 - 1$

$2y^2 = 2$

$y = \pm 1$



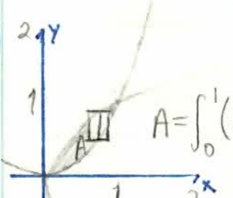
$A = \int_{-1}^{1} 1 - y^2 - (y^2-1)\, dy$

$= \int_{-1}^{1} 2 - 2y^2 dy$

$= 2x - \frac{2}{3}y^3 \Big|_{-1}^{1}$

$= 2 - 2/3 - [-2 + 2/3] \quad 4 - 4/3 = \underline{8/3}$

7)



$A = \int_0^1 (\sqrt{x} - x^2)dx$

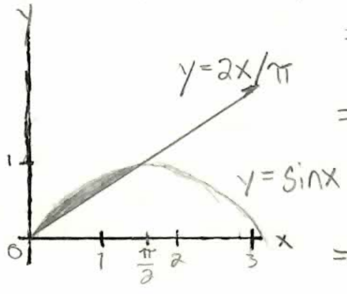
$[2/3\, x^{(3/2)} - 1/3\, x^3]_0^1$

$y = x^2,\ y^2 = x$

$y = \pm\sqrt{x}$

$= 2/3 - 1/3 = 1/3$

12) $y = \sin x,\ y = 2x/\pi,\ x \geq 0$



$= \int_0^{\pi/2} \sin x - (2x/\pi)dx$

$= \int_0^{\pi/2} \sin x dx - \frac{2}{\pi}\int_0^{\pi/2} x dx$

$= -\cos x \Big]_0^{\pi/2} - \frac{2}{\pi}\Big[\frac{x^2}{2}\Big]_0^{\pi/2}$

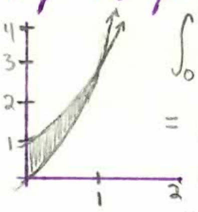
$= 0 + 1 - \frac{2}{\pi}\Big[\frac{\pi^2}{8}\Big]$

$= \underline{1 - \pi/4}$

15) $y = e^x,\ y = xe^x,\ x = 0$

$u = x \quad dv = e^x dx$

$du = 1dx \quad v = e^x$



$\int_0^1 e^x - (xe^x)dx$

$xe^x - \int e^x dx$

$= e^x\Big]_0^1 - [e^x(x-1)]_0^1$

$xe^x - e^x$

$e - 1 - 0 - 1 = e - 2$

$1 \cdot e - e = 0$

$e^0(-1) = -1$

$= e^x - xe^x + e^x\Big]_0^1$

$= 2e^x - xe^x\Big]_0^1$

$2e - e - 2 - 0$

$= \underline{e-2}$

23, 27, 28, 29-39

Chapter 6.1

Skip Lester
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19) $y = x\sin(x^2), y = x^4, a = .896$

$\int_0^a x\sin(x^2) - (x^4)dx$

$\Rightarrow$ let $u = x^2 \; du = 2x dx$

$= \frac{1}{2}\int_0^a \sin u du - \int_0^a x^4 dx$

$= \left. \frac{-\cos x^2}{2} - \frac{x^5}{5} \right]_0^a \approx$

$\frac{-\cos a^2}{2} - \frac{a^5}{5} + \frac{1}{2} \approx \underline{.037}$

21) $y = x^2\ln(x), y = \sqrt{x-1} \quad a \approx 1.3821$

$\int_1^{1.3821} \sqrt{x-1} - [x^2 \ln x] \, dx$

$= 2/3(x-1)^{3/2} - [1/3 x^3 \ln x - 1/9 x^3]_1^a$

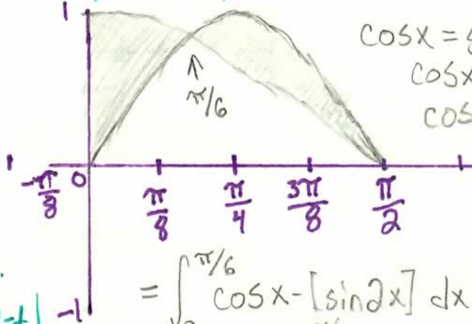
$= .05$

23) Sketch the region that lies between the curves $y = \cos x$ and $y = \sin 2x$ and between $x = 0$ and $x = \pi/2$. Notice this region consists of two seperate parts. Find the area of the region.

$\sin 2x = 2\sin x \cos x$

$u = 2x$
$du = 2dx$

$\int \sin 2x = \frac{1}{2}\int \sin u du$
$= -\frac{1}{2}\cos 2x$



$\cos x = \sin 2x$
$\cos x - \sin 2x = 0$
$\cos x - 2\sin x \cos x = 0$
$\cos x(1 - 2\sin x) = 0$

$\cos x = 0$
$1 = 2\sin x$

$x = \sin^{-1}(1/2) = \pi/6$

$= \int_0^{\pi/6} \cos x - [\sin 2x] \, dx \quad + \int_{\frac{\pi}{6}}^{\pi/2} \sin 2x - \cos x dx$

$= \int_0^{\pi/6} \cos x dx - \int_0^{\pi/6} \sin 2x \, dx + \left[\int_{\pi/6}^{\frac{\pi}{2}} \sin 2x dx - \int_{\pi/6}^{\pi/2} \cos x \, dx \right]$

$= \left[[\sin x]_0^{\pi/6} - [\frac{-1}{2}\cos 2x]_0^{\pi/6} \right] + \left[[-\frac{1}{2}\cos 2x]_{\frac{\pi}{6}}^{\frac{\pi}{2}} - [\sin x]_{\frac{\pi}{6}}^{\pi/2} \right]$

$\left[[1/2 - 0] - [-1/4 + 1/2] \right] + [1/2 + 1/4] - [1 - 1/2]$

$1/2 + 1/4 - 1/2 + 1/2 + 1/4 - 1/2 = \underline{1/2}$

27) The widths (in meters) of a kidney-shaped pool were measured at two meter intervals as indicated in the figure. $(\Delta x = 2)$ Simpson's Rule that shit!

n=9 is odd!

WTF?

We are only given 7 measures...
so I add ~~two~~ Zero samples

$\frac{\Delta x}{3}(1+4+2+4+2+4+2+4+1)$

$= 2/3(0 + (4 \cdot 6.2) + (2 \cdot 7.2) + (4 \cdot 6.8) + (2 \cdot 5.6) + (4 \cdot 5) + (2 \cdot 4.8) + (4 \cdot 4.8) + 0) = 2/3(126.4) \approx \underline{84 \, m^2}$

28) Given 11 measurements of the thickness of a wing (in cm), taken at 20 cm intervals, Use Simpson's to estimate the area. $\Delta x = 20$. n $\{\overset{1}{5.8}, \overset{4}{20.3}, \overset{2}{26.7}, \overset{4}{29.0}, \overset{2}{27.6}, \overset{4}{27.3}, \underset{2}{23.8}, \underset{4}{20.5}, \underset{2}{15.1}, \underset{4}{8.7}, \underset{1}{2.8}\}$

$= \frac{20}{3}[5.8 + (4 \cdot 20.3) + (2 \cdot 26.7) + (4 \cdot 29) + (2 \cdot 27.6)$

$+ (4 \cdot 27.3) + (2 \cdot 23.8) + (4 \cdot 20.5) + (2 \cdot 15.1)$

$+ (4 \cdot 8.7) + 2.8] \approx \underline{4121.\overline{3} \, m^2}$

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29-39 odd

29) If the birth rate of a population is given by $b(t)=2200e^{0.024t}$ people per year and the death rate is $d(t)=1460e^{0.018t}$ people per year, find the area between these curves for $0 \le t \le 10$. What does this area represent?

• For $0 \le t \le 10$, $b(t) > d(t)$

$\therefore A = \int_0^{10} b(t)-d(t)\,dt$

$\int e^{.024t} \Rightarrow \int e^u\, du/.024$

$u=.024t \quad du=.024dt$

$dt = du/.024$

$$= \int_0^{10} 2200e^{.024t} - 1460e^{.018t}\,dt = 2200\int_0^{10} e^{.024t}dt - 1460\int_0^{10} e^{.018t}dt$$

$$= \left[\frac{2200}{.024}\left(e^{.024t}\right) - \frac{1460}{.018}\left(e^{.018t}\right)\right]_0^{10} \approx$$ 8868 people, representing the increase in population.

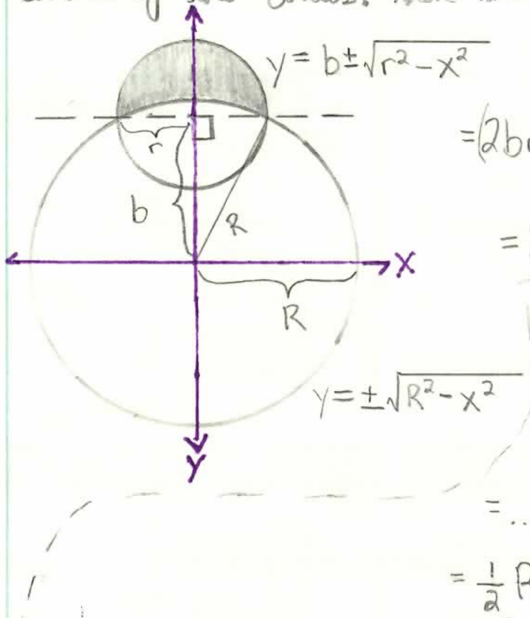
$x^2+(y-b)^2=r^2$

$(y-b)^2=r^2-x^2$

$y-b=\sqrt{r^2-x^2}$

$y=b+\sqrt{r^2-x^2}$

31) Find the area of the crescent-shaped region bounded by the arcs of circles with radii r and R. Let the equation of the large circle be $x^2+y^2=R^2$. Then the equation of the smaller circle is $x^2+(y-b)^2=r^2$, where $b=\sqrt{R^2+r^2}$, the distance between the centers of the circles. Then the area is $\int_{-r}^{r}\left[b+\sqrt{r^2-x^2}\right]-\left[\sqrt{R^2-x^2}\right]dx$



even symmetry $\rightarrow 2\int_0^r b\,dx + \underbrace{2\int_0^r \sqrt{r^2-x^2}\,dx}_{\text{area of } 1/2 \text{ circle}} - 2\int_0^r \sqrt{R^2-x^2}\,dx$

$$=(2br)+\underset{\text{area of half circle}}{\left(\pi r^2/2\right)} - 2\int_0^r \sqrt{R^2-x^2}\,dx \Big\}$$ let $x = R\sin\theta$, $dx = R\cos\theta\, d\theta$

$\theta = \sin^{-1}(x/R)$

$$= 2r\sqrt{R^2+r^2} + \pi r^2/2 - 2\int_0^r \sqrt{R^2-x^2}\,dx \Rightarrow \ldots \int_0^r \sqrt{R^2-R^2\sin^2\theta}\,dx$$

$$=\ldots \int_0^r \sqrt{R^2(1-\sin^2\theta)}(R\cos\theta)\,d\theta$$

$$=\ldots \int_0^r \sqrt{R^2\cos^2\theta}\,(R\cos\theta)\,d\theta$$

$\cos(\sin^{-1}(x/R)) = \sqrt{\left(1-\frac{x^2}{R^2}\right)}$

$1-\frac{x^2}{R^2}$

$R^2-\frac{x^2}{R^2}$

$\sqrt{1-\frac{x^2}{R^2}}$

$=\frac{\sqrt{R^2-x^2}}{\sqrt{R^2}}$

$$=\ldots \int_0^r R^2\cos^2\theta\, d\theta = \ldots \int_0^r R^2(1/2)(1+\cos\theta)\,d\theta$$

$$=\frac{1}{2}R^2\int(1+\cos 2\theta)\,d\theta \Rightarrow 1/2R^2(\theta + ((1/2)\sin 2\theta)$$

$$=\frac{R^2}{2}\left[(\theta+\sin\theta\cos\theta)\right] = \frac{R^2}{2}\left[\sin^{-1}\left(\frac{x}{R}\right)+(x/R)\left(\sqrt{1-x^2/R^2}\right)\right.$$

$$=\frac{R^2}{2}\sin^{-1}\left(\frac{x}{R}\right)+\left(\frac{\cancel{R^2}}{2}\cdot\frac{x}{\cancel{R}}\ \frac{\sqrt{R^2-x^2}}{\cancel{R}}\right)$$

$$=\ldots -2\left[\frac{R^2}{2}\sin^{-1}\left(\frac{x}{R}\right)+\left(\frac{x}{2}\cdot\sqrt{R^2-x^2}\right)\right]_0^r$$

$$= -R^2\sin^{-1}(r/R) - r\sqrt{R^2-r^2}$$

$$2r\sqrt{R^2+r^2}+\frac{\pi r^2}{2}-R^2\sin^{-1}\left(\frac{r}{R}\right)-r\sqrt{R^2-r^2}$$

$$= r\sqrt{R^2+r^2}+\frac{\pi r^2}{2}-R^2\sin^{-1}\left(\frac{r}{R}\right)$$

33-39odd

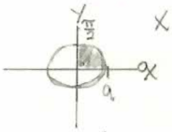
Skip Lester Calculus II
February 9th 2013 Chapter 6.1

33) Use the parametric equations of an ellipse to find the area it encloses.

$x = a\cos\theta$ $dx = -a\sin\theta\, d\theta$ $0 \le \theta \le 2\pi$ • an ellipse is symmetric w.r.t x & y axes •

$y = b\sin\theta$ • We therefor consider the area in quadrant I. •

$$A = 4\int_0^a y\,dx = 4\int_{\frac{\pi}{2}}^{0} b\sin\theta(-a\sin\theta)\,d\theta$$



$x = 0 \Rightarrow \theta = \pi/2$
$x = a \Rightarrow \theta = 0$

•flipped limits of integration

$$= 4ab\int_0^{\pi/2} \sin^2\theta\, d\theta = 2ab\int_0^{\pi/2}(1-\cos 2\theta)\,d\theta = 2ab\left[\theta - \left(\tfrac{1}{2}\cdot\sin 2\theta\right)\right]_0^{\pi/2} = \underline{\pi ab}$$

35) Find the area enclosed by the x-axis and the curve $\{x = 1+e^t, y = t - t^2\}$

eliminated the parameter

$$x - 1 = e^t \rightarrow t = \ln(x-1) \rightarrow y = \ln(x-1) - \ln(x-1)^2$$

$$= \int_2^{1+e} \ln(x-1)\,dx - \int_2^{1+e} \ln(x-1)^2\,dx$$

$0 = \ln(x-1) - \ln(x-1)^2$

$x = 2, x = 1+e$

$$= \left[x\ln(x-1) - \ln(x-1) - x\right]_2^{1+e} - \left[(x-1)(\ln(x-1)^2 - 2\ln(x-1) + 2\right]_2^{1+e}$$

$$1 \qquad - [-2 + e] = \underline{3 - e} \approx .281718 \text{ units}^2$$

37) Find the area bounded by the loop of the curve with parametric equations $\overset{f(t)}{x = t^2}$, $\overset{g(t)}{y = t^3 - 3t}$ $\rightarrow \int y\cdot x\,dt \Rightarrow \int_a^b y\,dx = \int_\alpha^\beta g(t)f'(t)$

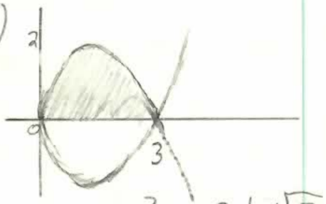
$dx = 2t\,dt$ $dy = 3t^2 - 3dt$

$t^3 - 3t = 0$ $\quad 2\int_0^3 y\,dx = 2\int_0^{-\sqrt{3}}(t^3 - 3t)2t\,dt$

$t(t^2-3) = 0$ $\quad = 4\int_0^{-\sqrt{3}} t^4 - 3t^2\,dt = 4\left[(t^5/5) - t^3\right]_0^{-\sqrt{3}}$

$t = 0$ $t^2 - 3 = 0$

$t = \pm\sqrt{3}$ $\quad = 4\left[\frac{1}{5}(-\sqrt{3})^5 - (-\sqrt{3})^3\right] = 24/5\sqrt{3} \approx 8.31 \text{ units}^2$



$x = 3, y = 0, t = \pm\sqrt{3}$

39) Find the values of c such that the area bounded by the parabolas $y = x^2 - c^2$ and $y = c^2 - x^2$ is 576. $c > 0$

$$= 4\int_0^c y\,dx = 4\int (c^2 - x^2)\,dx = 4\left[c^2x - \left(x^3/3\right)\right]_0^c$$

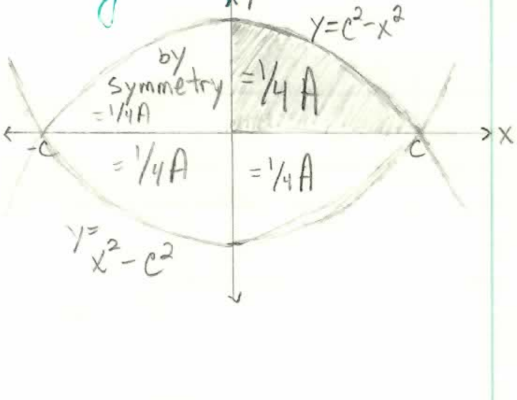
(x=c)

$$= 4\left[c^3 - c^3/3\right] = 4\left[2/3\, c^3\right] = 8/3\, c^3$$

$576 = 8/3\, c^3$

$3/8(576) = c^3$

$\underline{\sqrt[3]{216} = c = 6}$

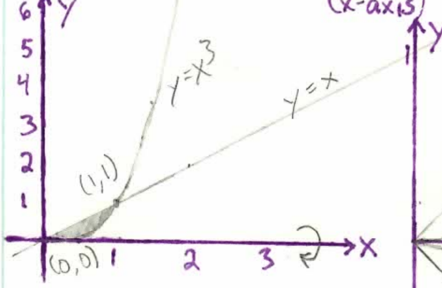


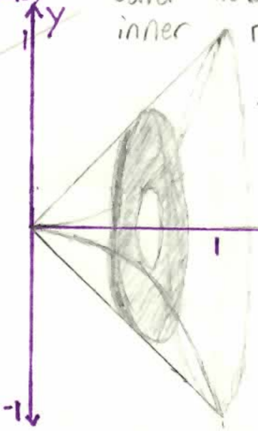
Find the volume of the solid obtained by rotating the region bounded by the given curves about the given line. Chapter 6.2 Volumes

Skip Lester 10 February 2013

5/5 perfect

5) $y=x^3$, $y=x$, $x\geq 0$, about $y=0$ (x-axis)





We use a washer

outer radius ⇒ $y=x$

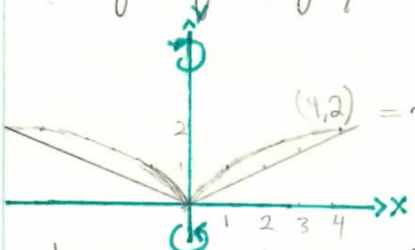
inner radius ⇒ $y=x^3$

$$\therefore A(x)=\pi(x^2)-\pi(x^3)^2$$
$$=\pi(x^2-x^6)$$
$$V(x)=\int_0^1\pi(x^2-x^6)\,dx$$
$$=\pi\left[\,x^3/3-x^7/7\right]_0^1 \qquad 7/21-3/21=4/21$$
$$=\pi(1/3-1/7)=\underline{(4\pi)/21\text{ units}^3}$$

7) $y^2=x$, $x=2y$; about y

Integrating along y; $A(y)=\pi(2y)^2-\pi(y^2)^2$

$$=\pi(4y^2-y^4)$$



$$=\pi\int_0^2 4y^2-y^4$$

160 − 96 = 64

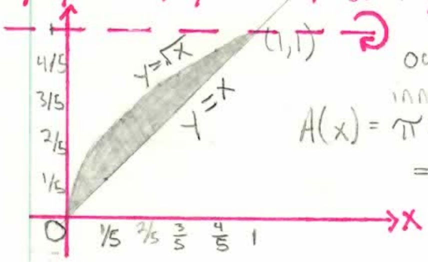
$$=\pi\cdot 4\int_0^2 y^2dy-\left[\pi\cdot\int_0^2 y^4dy\right.$$

←

outer ⇒ $x=2y$ inner ⇒ $x=y^2$

$$=\frac{4\pi y^3}{3}-\frac{\pi y^5}{5}\Big]_0^2=\frac{32\pi}{3}-\frac{32\pi}{5}=\boxed{\frac{64\pi}{15}}$$

9) $y=1x$; $y=\sqrt{x}$; about $y=1$



outer = $y=x$

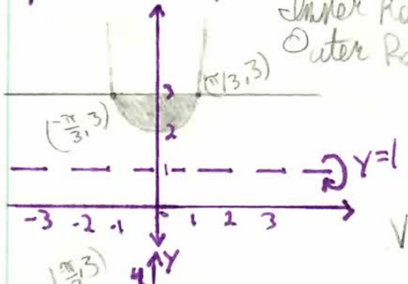
inner = $y=x^{1/2}$

$$A(x)=\pi(x)^2-\pi(x^{1/2})^2$$
$$=\pi(x^2-x)$$
$$V(x)=\pi\int_1^0 x^2-x\,dx$$
$$=\pi(x^3/3-x^2/2)\Big]_1^0$$
$$=-\left[\pi/3-\pi/2\right]=-2\pi/6+3\pi/6=\boxed{\pi/6}$$

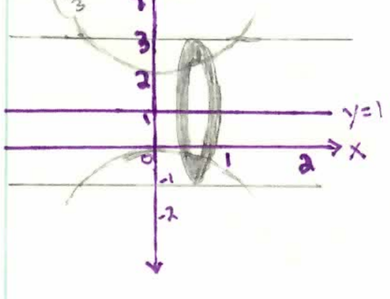
11) $y=1+\sec x$, $y=3$; about $y=1$

Inner Radius = $1+\sec x-1=\sec x$

Outer Radius = $3-1=2$



$$A(x)=\pi 2^2-\pi\sec^2x$$
$$=\pi(4-\sec^2x)$$
$$V(x)=\int_{-\pi/3}^{\pi/3}\pi(4-\sec^2x)$$

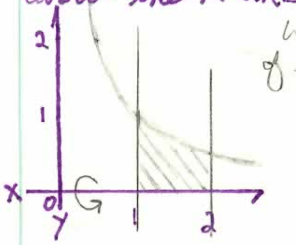


Symmetry

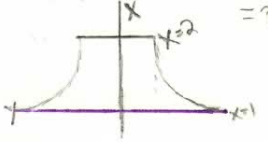
$$=2\pi\int_0^{\pi/3}4-\sec^2x$$
$$=2\pi\left[4x-\tan x\right]_0^{\pi/3}$$
$$=2\pi\left[4\pi/3-\sqrt{3}\right]=\boxed{8\pi^2/3-2\sqrt{3}\,\pi}$$

6.2 Volumes of Solids   Skip Lester February 10, 2013

13) $y = 1/x, x = 1, x = 2, y = 0$; about the x-axis



We use a disk of radius $1/x$



$A(x) = \pi(1/x)^2$

$V = \int_1^2 \pi \, 1/x^2 \, dx$

$= \pi \int_1^2 1/x^2 \, dx \quad x^{-2} \to -x^{-1}$

$= \pi \left[-1/x \right]_1^2$

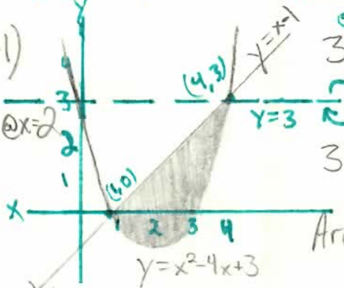
$= -\pi/2 + \pi = \underline{\pi/2}$

15) $x - y = 1, y = x^2 - 4x + 3$; about $y = 3$

$-y = 1 - x \quad y = (x-3)(x-1)$

$y = x - 1 \quad -(-4)/2(1) \Rightarrow$ vertex @ $x = 2$

We choose a washer.



WASHER RADII

OUTER $3 - (x^2 - 4x + 3)$

Inner $3 - (x - 1)$

$V_{Shape} = \pi \int_1^4 [3 - (x^2 - 4x + 3)]^2 - [3 - (x - 1)]^2 dx$

$= \pi \int_1^4 (4x - x^2)^2 - (4 - x)^2 dx$

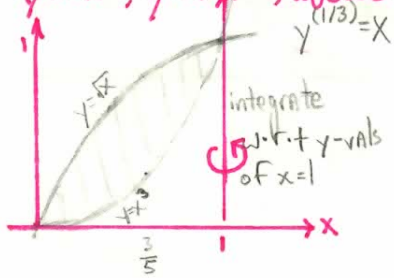
$= \pi \int_1^4 16x^2 - 8x^2 + x^4 - 16 + 8x - x^2 \, dx$

$Area_{(Washer)} = \pi [3 - (x^2 - 4x + 3)]^2 - \pi [3 - (x - 1)^2]$

$= \pi \int_1^4 (x^4 - 8x^3 + 15x^2 + 8x - 16) dx$

$= \pi \left[\frac{x^5}{5} - 2x^4 + 5x^3 + 4x^2 - 16x \right]_1^4 = \boxed{\left(\frac{108}{5}\right)\pi}$

17) $y = x^3, y = x^{(1/2)}$; about $x = 1$

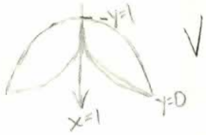


$y^{(1/3)} = x, \; y^2 = x$

inner radius of washer || outer

$\pi(1 - (\sqrt[3]{y}))^2 \quad || \quad \pi(1 - y^2)^2$

$A(x) = \pi((1 - y^2)^2 - (1 - \sqrt[3]{y})^2)$



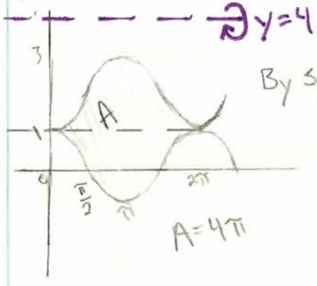
$V = \pi \int_0^1 1 - 2y^2 + y^4 - (1 - 2y^{(1/3)} + y^{(2/3)}) dy$

$= \pi \int_0^1 y^4 - y^{2/3} - 2y^2 + 2y^{1/3} dy$

$= \pi \left[\frac{y^5}{5} - \frac{3}{5} y^{5/3} - \frac{2}{3} y^3 + \frac{3}{2} y^{4/3} \right]_0^1 = \boxed{\frac{13\pi}{30}}$

20) Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified line

$y = \cos x, y = 2 - \cos x, 0 \le x \le 2\pi; \overset{\text{about}}{y = 4}$



By symmetry, $A = 2\int_0^{2\pi} (1 - \cos x) dx$

$= 2[x - \sin x]_0^{2\pi}$

$= 2[2\pi - 0 - 0 - 0]$

$= 4\pi$

∴ Using a washer and integrating along x, we have

inner radius $\Rightarrow 4 - [2 - \cos x] = \cos x + 2$

outer radius $\Rightarrow 4 - \cos x =$

$V = \pi \int_0^{2\pi} (4 - \cos x)^2 - [(\cos x + 2)^2] dx$

$V = \pi \int_0^{2\pi} (12 - 12 \cos x) dx$

$V = 12\pi \int_0^{2\pi} (1 - \cos x) dx$

$= 12\pi [x - \sin x]_0^{2\pi}$

$24\pi^2$

21) Use mathematica to find the volume:

$\overset{f(x)}{y=2+x^2\cos x}$, $\overset{g(x)}{y=x^4+x+1}$; about x-axis

intersects : $x=-1.28786$, $x=.88425$

$y=2.46304$, $y=2.49562$

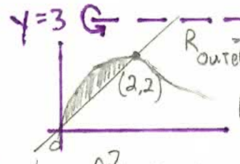
$f(x)>g(x)$ on this interval $(-1.28, .88)$

$A=\pi f(x)^2-\pi g(x)^2$

$$V=\pi\int_{-1.28}^{.88}[2+x^2\cos x]^2-[x^4+x+1]^2dx \approx \underline{23.7802}$$

24) Use mathematica to find volume:

$y=x$, $y=xe^{1-x/2}$; about $y=3$



$R_{outer}=\pi x^2$

$R_{inner}=\pi(xe^{1-x/2})^2$

$$V=\pi\int_0^2(x^2-(xe^{1-\frac{x}{2}})^2)dx=\underline{\pi(38/3-2e^2)}$$

27) a cat-scan produces equally spaced cross-sectional views of a human organ that provide information about the organ that could otherwise be obtained only by surgery. Suppose that a cat-scan of a human liver show cross sections spaced 1.5 cm. apart. The liver is 15 cm. long, and the cross sectional areas, in cm², are 0, 18, 58, 79, 94, 106, 117, 128, 63, 39, and 0. Use midpoint to estimate the volume of this liver. 11 samples

Take $n=10/2=5$ $V=\int_0^{15}A(x)dx\approx M_5=\left[\frac{15}{5}(A(1.5)+\cdots\right]$

$=3(18+79+106+128+39)=3\cdot 370=1110\text{ cm}^3$, rough + sloppy

26, 28, 29-37 odd
40-43, 47, 48

Chapter 6.2 Skip Lester 11 February 2013

26) Each integral represents the shape obtained by gypsy magic. or it represents the volume of a solid. Describe the solid / gypsy magic shape.

a) $\pi\int_2^5 y\,dy$ a cone

a solid obtained by rotating the line $x=\sqrt{y}$ about the x-axis, whose end points are $y=2$ & $y=\sqrt{5}$.

b) $\pi\int_0^{2\pi/4}[(1+\cos x)^2-1^2]dx$

a hollow cylinder whose outer radius is given by $y=1+\cos x$ and whose inner radius is $y=1$. The cylinder is $\pi/2$ units long.

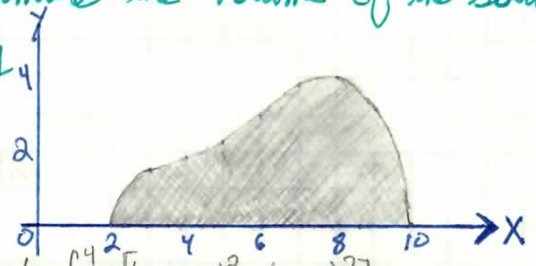
28) a log 10m long is cut at 1 meters intervals and its cross sectional area A (at a distance x from the log) are listed in the table.
Use the midpoint rule with $n=5$ to estimate the volume of the log.

		1		2		3		4		5	
X(m)	0	1	2	3	4	5	6	7	8	9	10
A(m²)	.68	.65	.64	.61	.58	.59	.53	.55	.52	.50	.48

$\Delta x = 10/n \rightarrow 10/5 = 2$

$M_5 = 2[.65+.61+.59+.55+.50] \approx 5.8\ m^3$

29) a) If the region shown in the figure is rotated about the x-axis to form a solid, use Simpson's rule to estimate the volume of the solid with a) $n=8$ about x-axis and b) $n=4$ about y-axis



$V=\int_2^{10}\pi[f(x)]^2dx$

a) $b-a=8$ $\Delta x = b-a/n = 8/8 = 1$

$S_8 = \pi/3[0^2+4(1.6^2)+2(1.9^2)+4(2.2^2)+2(3^2)$
$+4(3.8^2)+2(4.0^2)+4(3.1)^2+0^2\ ]$

$= \pi/3(183.02) = 191.65$ units^3
(Disk method)

b) $V=\int_0^4 \pi[(\text{outer})^2-(\text{inner})^2]dy$

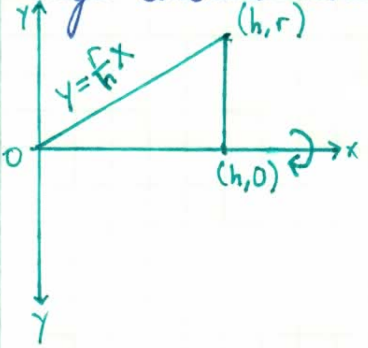
$\Delta x = \frac{b-a}{n} = 4/4 = 1$

$= \pi/3[(10^2-2^2)+4(9.8^2-2.5^2)+2(9.5^2-4.3^2)$
$+4(9.2^2-6^2)+(8^2-8^2)]$

$= \pi/3[793.24] \approx 830.67$ units^3

Find the volume of the described solid S.

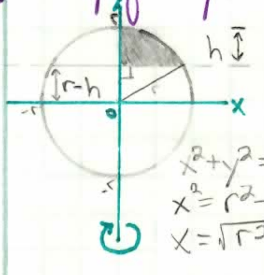
31) a right circular cone of height h and base radius r.



Using disks, $V = \pi\int y^2 dx \rightarrow V = \pi\int_0^h [(r/h)x]^2 dx$

$$= \pi\int_0^h \frac{r^2}{h^2}x^2 = \frac{\pi r^2}{h^2}\left[\frac{x^3}{3}\right]_0^h \Rightarrow \frac{\pi r^2}{h^2}\cdot\frac{h^3}{3} - 0 = \boxed{\frac{\pi r^2 h}{3}}$$

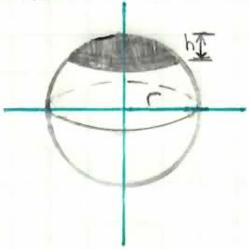
★ 33) a cap of a sphere with radius r and height h



$x^2 + y^2 = r^2$

$x^2 = r^2 - y^2$

$x = \sqrt{r^2 - y^2}$



$V = \pi\int x\,dy$

$$V = \pi\int_{r-h}^{r}\left[\sqrt{r^2-y^2}\right]^2 dy = \pi\left[r^2 y - \frac{y^3}{3}\right]_{r-h}^{r}$$

$$V = \pi\int_{r-h}^{r}(r^2 - y^2)dy$$

$$= \pi\left[r^3 - \frac{r^3}{3}\right] - \pi\left[r^2(r-h) - \frac{(r-h)^3}{3}\right]$$

$$= \pi\left\{\frac{2r^3}{3} - \frac{(r-h)}{3}\left[3r^2 - (r-h)^2\right]\right\}$$

$$= \pi\left\{\frac{2r^3}{3} - \frac{(r-h)}{3}\left[3r^2 - \left[r^2 - 2rh + h^2\right]\right]\right\}$$

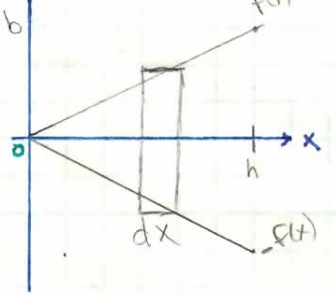
$$= \frac{\pi}{3}\left\{2r^3 - (r-h)(2r^2 + 2rh - h^2)\right\}$$

$$= \frac{\pi}{3}\left(\cancel{2r^3} - \left[\cancel{2r^3} + \cancel{2r^2h} - rh^2 - \cancel{2r^2h} - 2rh^2 + h^3\right]\right)$$

$$= \frac{\pi}{3}\left(0 - (-3rh^2 + h^3)\right)$$

$$= \pi/3(3rh^2 - h^3) = \underline{\frac{\pi h^2}{3}(3r - h)}$$

35) a pyramid with height h and rectangular base with dimensions b and 2b.

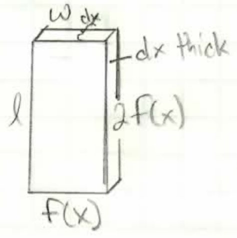


- we place the pyramid on its side so it's vertex lies at origin and consider one slice

$f(x) = \frac{b}{h}\cdot x \quad V = \int 2\left[\left(\frac{b}{h}\right)\cdot x\right]^2 dx$

$$= \frac{b^2}{h^2}\int 2x^2 dx = \frac{b^2}{h^2}\left[\frac{2x^3}{3}\right]_0^h$$

$$= \frac{b^2}{h^2}\cdot\frac{h^3}{3}\cdot 2 = \boxed{\frac{2}{3}b^2 h}$$

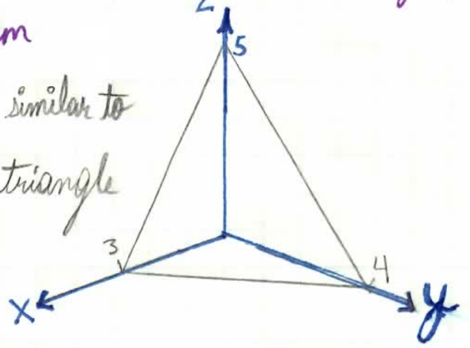


$\text{Area} = 2F(x)\cdot F(x)$

$= 2F(x)^2$

$\therefore V = \int_0^h 2f(x)^2 dx$

37) A tetrahedron with 3 mutually perpendicular faces and 3 mutually perpendicular edges with lengths 3 cm, 4 cm, and 5 cm



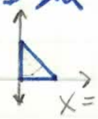
A cross-section at height Z is a triangle similar to the base. We multiply the lengths of the base triangle by proportionality factor of $\frac{5-Z}{5}$.

Then the triangle at height Z has area

$A(Z) = \frac{1}{2} \cdot 3\left(\frac{5-Z}{5}\right) \cdot 4\left(\frac{5-Z}{5}\right) = 6\left(1-\frac{Z}{5}\right)^2 \therefore V = \int_0^5 A(Z)\,dZ$

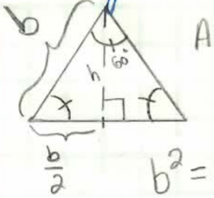
$= 6\int_0^5 \left(1-\frac{Z}{5}\right)^2 dZ \;\Big\|\; \begin{matrix}\text{let } u = 1-Z/5 \\ du = -1/5\,dZ\end{matrix} = 6\int_1^0 -u^2 \cdot 5\,du = 6\left[\frac{-5u^3}{3}\right]_1^0 = \frac{30}{3} = \underline{10\text{ cm}^3}$

40) The base of S is a triangular region with vertices (0,0), (1,0), and (0,1). Cross sections ~~to~~ perpendicular to the y-axis are equilateral triangles.



$y = 1-x$

$x = 1-y = b$



$A = = \frac{1}{2} b \cdot h$

$A = \frac{1}{2} b$

$b^2 = \left(\frac{b}{2}\right)^2 + h^2$

$b^2 - \frac{b^2}{4} = h^2$

$h^2 = 3b^2/4 \Rightarrow h = \sqrt{3}b/2$

$dV = \frac{1}{2} \cdot b \cdot \frac{\sqrt{3}b}{2}$

$dV = \frac{\sqrt{3}b^2}{4}$

$\therefore V = \int_0^1 \frac{\sqrt{3}}{4}(1-y)^2 dy$

let $u = 1-y$

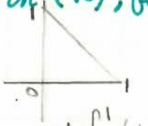
$du = -dy$

$= \frac{\sqrt{3}}{4}\int_1^0 u^2 - du$

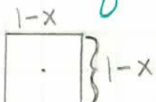
$= -\frac{\sqrt{3}}{4}\left[\frac{u^3}{3}\right]_1^0 = 0 + \frac{\sqrt{3}}{12}$

$\underline{\frac{\sqrt{3}}{12}\text{ units}^3}$

41) The same base S as in (40), but cross-sections perpendicular to the x-axis are squares.



$y = 1-x$



$A(x) = (1-x)^2 = 1-2x+x^2$

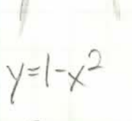
$V = \int_0^1 (1-2x+x^2)\,dx$

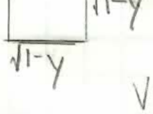
$= x - x^2 + x^3/3 \Big|_0^1$

$= 1-1+1/3-0$

$= \underline{1/3}\text{ units}^3$

42) The base of S is the region enclosed by the parabola $y = 1-x^2$ and the x-axis. Cross sections perpendicular to the y-axis are squares.





$y = 1-x^2$

$x^2 = 1-y$

$x = \sqrt{1-y}$

$A = 1-y$

$V = \int_{-1}^1 (1-y)\,dy$

$V = \left[y - y^2/2\right]_{-1}^1$

$V = 1 - 1/2 - [-1-1/2]$

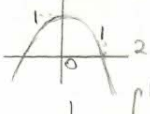
$1/2 + 3/2 = 2$

$\underline{2\text{ units}^3}$

43, 47, 48

Chapter 6.2 Skip Lester 2/13/13

43) The base of S is a region enclosed by $y=1-x^2$ and the x-axis. Cross sections perpendicular to the x-axis are isosceles triangles with height equal to the base.



$A(x) = ½ b \cdot h = ½ b^2$

$A = \frac{1}{2}\left[1-x^2\right]^2 \therefore V = \int_{-1}^{1} \frac{1}{2}\left[1-x^2\right]^2 dx = \frac{1}{2} \cdot 2\int_0^1 (1-x^2)^2 dx = \int_0^1 1-2x^2+x^4$

$= \left[x - \frac{2}{3}x^3 + \frac{x^5}{5}\right]_0^1 = \frac{15-10+3}{15} = \underline{8/15 \text{ units}^3}$

47) a) Cavalieri's Principle states that if a ~~that~~ family of parallel planes gives equal cross-sectional areas for two solids S_1 and S_2, then the volumes of S_1 and S_2 are equal. Prove it.

$V[S_1] = V[S_2] = \int_0^h A(z)dz$, where Z is the vertical axis in the x-y-z plane.

A(z) is linear with z for paralell lines.

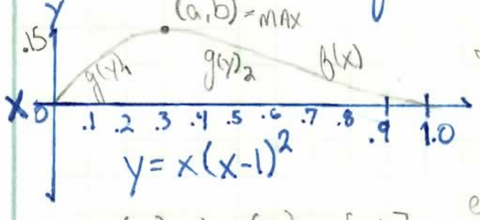
b) Use Cavalieri's Principle to find the volume of the ofligue cylinder

$V = \pi r^2 h$ because the cross-sectional area is the same as a right cylinder.

48) Find the volume common to two circular cylinders, each with radius r, if the axes of the cylinders intersect at right angles.

Chapter 6.3 Skip Lester 2/13/13 2.5/5

1) Let S be the solid obtained by rotating the region shown about the y-axis. Why is it awkward to use slicing. Sketch an approximating shell. What are its circumference and height? Use shells to find V.

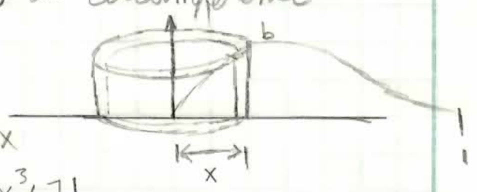


If we try washers, we have to solve $f(x)$ in terms of y, which would require two equations $g(y)_1$ and $g(y)_2$. We would then use

$g(y)_2 > g(y)_1$ on $[0,b]$

$$V = \pi\int_0^b [g(y)_2]^2 - [g(y)_1]^2 \, dy$$

But shells are easier: a shell has radius x, so its circumference is $2\pi x$. Its height is given by $y = x(x-1)^2$, so



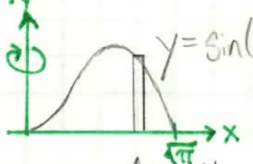
$$V = \int_0^1 2\pi x[(x)(x-1)^2]\,dx = 2\pi\int_0^1 x^2(x^2-2x+1)\,dx$$

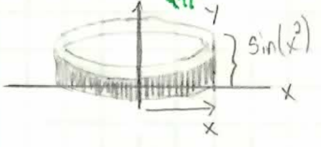
$$= 2\pi\int_0^1 [x^4 - 2x^3 + x^2]\,dx = 2\pi\left[\frac{x^5}{5} - \frac{x^4}{2} + \frac{x^3}{3}\right]_0^1$$

$$= 2\pi\left[\underset{6/30}{1/5} - \underset{15/30}{1/2} + \underset{10/30}{1/3}\right] = 2\pi \cdot 1/30 = \underline{\pi/15 \text{ units}^3}$$

✓

2) Let S be the solid obtained by rotating the region shown about the y-axis. Sketch a shell. Find the volume. Better than slicing? Why?





$\therefore$ circumference $= 2\pi x$

height $= F(x) = \sin(x^2)$

thickness $= dx$

$$V = \int_0^{\sqrt{\pi}} 2\pi x \sin(x^2)\,dx$$

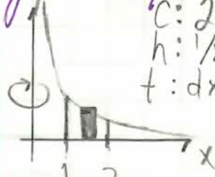
$$V = \pi\int_0^{\sqrt{\pi}} 2x \sin x^2 \, dx \quad \text{let } u = x^2 \quad du = 2x\,dx$$

$$V = \pi\int_0^{\pi} \sin u \, du = \pi[1 - -1]$$

$$V = \pi[-\cos u]_0^{\pi} \qquad = \underline{2\pi \text{ units}^3}$$

3-7 Use shells to find the volume of the shape obtained by rotating the region bounded by the given curves about the y-axis. Sketch a shell.

3) $y = 1/x$, $y = 0$, $x = 1$, $x = 2$



C: $2\pi x$

h: $1/x$

t: dx

$$V = \int_1^2 2\pi x(1/x)\,dx$$

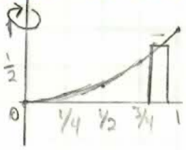
$$= 2\pi\int_1^2 dx$$

$$= 2\pi x\Big|_1^2$$

$$= (2\cdot 2\pi) - (1\cdot 2\pi) = \underline{2\pi}$$

✓

4) $y = x^2$, $y = 0$, $x = 1$



C: $2\pi x$

h: x^2

$$V = \int_0^1 2\pi x(x^2)\,dx$$

$$V = 2\pi\int_0^1 x^3\,dx$$

$$V = 2\pi\left[\frac{x^4}{4}\right]_0^1$$

$$V = 2\pi(1/4 - 0/4)$$

$$\underline{V = \pi/2}$$

Chapter 6.3 Shells Skip Lester 2/13/2013

5) $y=e^{-x^2}, y=0, x=0, x=1;$ rotated about y-axis

circumference $=2\pi x$

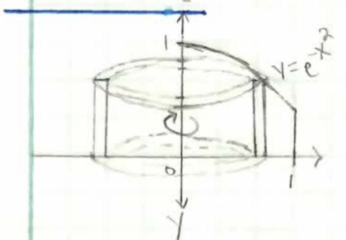
height $= e^{-x^2}$, thickness $= dx$

$V=\int_0^1 e^{-x^2}\cdot 2\pi x\, dx$

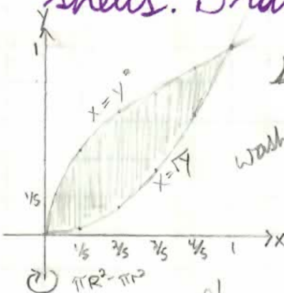
let $u=x^2$ $du=2x dx$

$V=\pi\int_0^1 e^{-u} du = \pi\left[-e^{-u}\right]_0^1$

$V=\pi\left[-1/e+1\right]$



8) Let V be the volume of the solid obtained by rotating about the y-axis the region bounded by $y=\sqrt{x}$ and $y=x^2$. Find V by slicing and shells. Draw stuff.



$\pi R^2-\pi r^2$

Slicing: $y=\sqrt{x}$ $y=x^2$
$x=y^2$ $x=\sqrt{y}$

washers

inner radius $\Rightarrow x=y^2$
outer $\Rightarrow x=\sqrt{y}$

$=\pi\int_0^1 [\sqrt{y}]^2-[y^2]^2\, dy$

$=\pi\int_0^1 y-y^4\, dy$

$=\pi\left[y^2/2 - y^5/5\right]_0^1$

$=\pi\left[1/2-1/5\right]=3\pi/10$

shells: C: $2\pi x$

height $=\sqrt{x}-x^2$

$=\int_0^1 2\pi x(\sqrt{x}-x^2)\,dx$

$=\pi\int_0^1 2(x^{3/2}-x^3)\,dx$

$=2\pi\left[2/5\, x^{5/2} - x^4/4\right]_0^1$

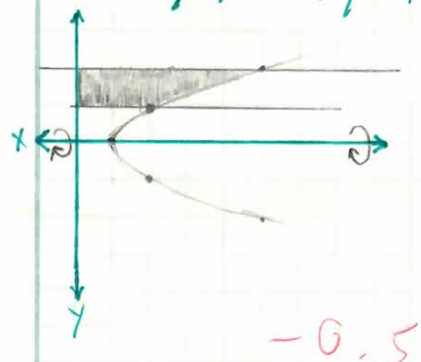
$=2\pi(2/5-1/4)$

$=4\pi/5-2\pi/4$

$=16\pi/20-10\pi/20=6\pi/20=3\pi/10$

9) Use shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the x-axis.

$x=1+y^2, x=0, y=1, y=2$ C: $2\pi x$



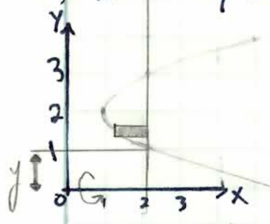
-0.5

missing 11, 13

Chapter 6.3

Skip Lester 14 February 2013

11) $x = 1 + (y-2)^2, x = 2$



the height $= 2 - [1 + (y-2)^2]$

$x = 1$

$= 1 - [y^2 - 4y + 4]$

$h = -y^2 + 4y - 3$

$C = 2\pi y \therefore V = 2\pi\int_1^3 y(-y^2+4y-3)dy$

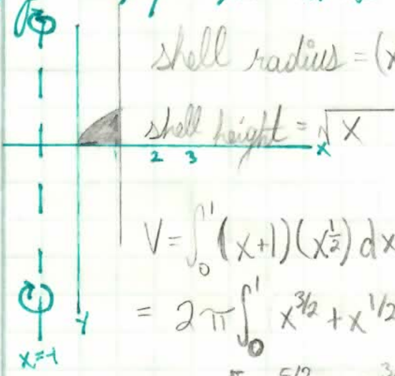
$= 2\pi\int_1^3 -y^3+4y^2-3y\,dy$

$= 2\pi[-y^4/4 + 4y^3/3 - 3y^2/2]_1^3$

$= 2\pi[[-81/4 + 36 - 27/2] - [-1/4 + 4/3 - 3/2]]$

$= \frac{16\pi}{3}$

14) $y = \sqrt{x}$, $y = 0$, $x = 1$, about $x = -1$



shell radius $= (x+1)$

shell height $= \sqrt{x}$

$V = \int_0^1 (x+1)(x^{\frac{1}{2}})dx$

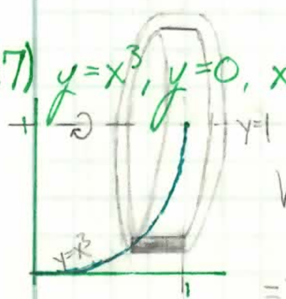
$= 2\pi\int_0^1 x^{3/2} + x^{1/2}dx$

$= 2\pi\left[\frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3}\right]_0^1$

$= 2\pi\left[\frac{2}{5} + \frac{2}{3}\right]$

$= 32\pi/15$ units^3

17) $y = x^3$, $y = 0$, $x = 1$, about $y = 1$



$r = (1-y)$ $h = (1 - y^{(1/3)})$

$V = 2\pi\int_0^1 (1-y)(1-y^{1/3})dy$

$= 2\pi\int_0^1 (1 - y - y^{1/3} + y^{4/3})dy$

$y = x^3$

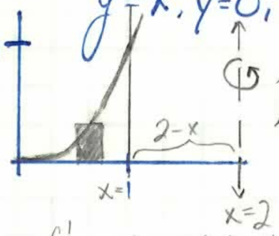
$\sqrt[3]{y} = x$

$= 2\pi[y - \frac{1}{2}y^2 - 3y^{4/3}/4 + \frac{3}{7}y^{7/3}]_0^1$

$= 2\pi[1 - 1/2 - 3/4 + 3/7] = \boxed{\frac{5\pi}{14}}$

13) Use shells to find volume

$y = x^4$, $y = 0$, $x = 1$; about $x = 2$



shell radius $= 2 - x$

height $= x^4$

$\therefore \int_0^1 2\pi(2-x)(x^4)dx$

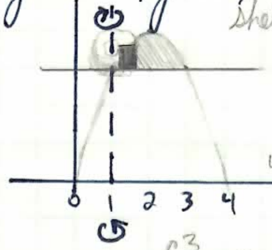
$= 2\pi\int_0^1 (2x^4 - x^5)dx$

$= 2\pi[\frac{2}{5}x^5 - x^6/5]_0^1$

$= 2\pi(\frac{2}{5} - 1/6) = 7/15\,\pi$

15) $y = 4x - x^2$, $y = 3$; about $x = 1$

height $= (4x - x^2) - 3$



Shell radius $= (x-1)$

$= \int 2\pi(x-1)(4x - x^2 - 3)dx$

$4x^2 - x^3 - 3x - 4x + x^2 + 3$

$-x^3 + 5x^2 - 7x + 3$

$= 2\pi\int_1^3 (-x^3 + 5x^2 - 7x + 3)dx$

$= 2\pi[-x^4/4 + 5x^3/3 - 7x^2/2 + 3x]_1^3$

$= 2\pi[(\frac{-81}{4} + 45 - \frac{62}{2} + 9) - (-1/4 + 5/3 - 7/2 + 3)$

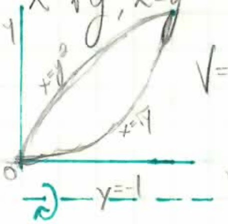
$= 2\pi(4/3) = 8/3\,\pi$

18) $y = x^2$, $x = y^2$ about $y = -1$

height $= \sqrt{y} - y^2$

$x = \sqrt{y}$, $x = y^2$

$r = (1 + y)$



$V = 2\pi\int_0^1 (1+y)[(\sqrt{y} - y^2)]dy$

$V = 2\pi\int_0^1 (\sqrt{y} - y^2 + y^{3/2} - y^3)dy$

$V = 2\pi[\frac{2x^{3/2}}{3} - y^3/3 + \frac{2y^{5/2}}{5} - y^4/4]_0^1$

$V = 2\pi[(2/3 - 1/3 + 2/5 - 1/4) - 0]$

$2\pi \cdot (1/3 + 2/5 - 1/4)$

$2\pi \cdot 29/60$

$58\pi/60 = 29\pi/30$

20,22,23,27

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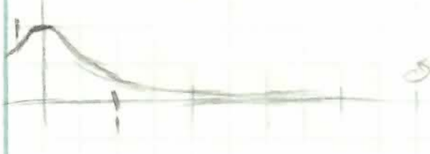
20) Set up an integral for the volume of the shape obtained by rotating the region bounded by the given curves about the specified line.

$y=e^{-x^2}$, $y=0$, $x=0$, $x=4$; about $x=5$

$V=2\pi\int(5-x)(e^{-x^2})\,dx$



22) S_{10} (pg. 454) $r=x$ $V=2\pi\int(x)F(x)dx$

$\Delta x = b-a/10 = 12-2/10 = 1$ $(S, x, f(x))$

$S_{10} \approx \frac{2\pi}{3}\Big[(1\cdot 2\cdot 0)+(4\cdot 3\cdot 2)+(2\cdot 4\cdot 2.7)$

$+(4\cdot 5\cdot 4)+(2\cdot 6\cdot 4.4)+(4\cdot 7\cdot 4)$

$+(2\cdot 8\cdot 2.6)+(4\cdot 9\cdot 2)+(2\cdot 10\cdot 1.6)$

$+(4\cdot 11\cdot 1)+(1\cdot 12\cdot 0)\Big]$

$\frac{2\pi}{3}[496.4] \approx \underline{331\pi}$

23) Describe the solid.

a) $\int_0^3 2\pi x^5 dx$ b) $\int_0^1 2\pi(3-y)(1-y^2)dy$

$=2\pi\int_0^3 x(x^4)dx$ $= 2\pi\int_0^1(3-y)(1-y^2)dy$

(a) the shape made by rotating the region bounded by $y=x^4$, $y=0$, $x=0$, & $x=3$ about the y-axis. (b) is the region bounded by $x=1-y^2$, $y=0$, $y=1$ rotated about $y=3$.

Chapter 6.4 {1,3,5,6,7,9-12,14,15,16} Skip Lester February 14, 2013

1) Use the arc length formula (2) to find the length of the curve $y=2x-5$, $-1 \leq x \leq 3$. Check w/ distance formula. $\frac{dy}{dx}=2$

$$L=\int_{-1}^{3}\sqrt{1+\left(\frac{dy}{dx}\right)^2}dx=\int_{-1}^{3}\sqrt{1+2^2}\,dx=\sqrt{5}\int_{-1}^{3}dx=[3--1]\cdot\sqrt{5}=4\sqrt{5}$$

$2(3)-5=1$

$2(-1)-5=-7$

$$D=\sqrt{[x_F-x_i]^2+[y_F-y_i]^2}=\sqrt{[-1-3]^2+[-7-1]^2}=\sqrt{16+64}=\sqrt{80}=4\sqrt{5}$$

3) Set up an integral that represents the length of the curve.

$y=\sin x$, $0 \leq x \leq \pi$

$\frac{dy}{dx}=\cos x$

$$L=\int_0^{\pi}\sqrt{1+\cos^2 x}\,dx$$

≈ 3.8202

5) ~~$x=t\cos t, y=t\sin t$~~

$x=t+\cos t$, $y=t-\sin t$ $0 \leq t \leq 2\pi$

$\frac{dx}{dt}=1-\sin t$ $\frac{dy}{dt}=1-\cos t$

$\left(\frac{dx}{dt}\right)^2=(1-\sin t)^2$ $\left(\frac{dy}{dt}\right)^2=(1-\cos t)^2$

$=[1-2\sin t+\sin^2 t]+[1-2\cos t+\cos^2 t]$

$\cdot \sin^2 t + \cos^2 t = 1 \cdot$

$2-2\sin t-2\cos t+1 \rightarrow 3-2\sin t-2\cos t$

$$L=\int_0^{2\pi}\sqrt{[3-2\sin t-2\cos t]}\,dt=10.0367 \text{ units}^3$$

6) $x=t\cos t$, $y=t\sin t$, $0 \leq t \leq 2\pi$

$\frac{dx}{dt}=\cos t-t\sin t$

$\frac{dy}{dt}=\sin t+t\cos t$

$$L=\int_0^{2\pi}\sqrt{(\cos t-t\sin t)^2+(\sin t+t\cos t)^2}\,dt$$

$L=20.4644$ unit

7) Find the exact length of the curve

$x=1+3t^2$ $y=4+2t^3$ $0 \leq t \leq 1$

$x'=6t$ $y'=6t^2$

$$L=\int_a^b\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}dt \Rightarrow \int_0^1\sqrt{(6t)^2+(6t^2)^2}\,dt$$

$$=\int_a^b\sqrt{6t^2+6t^4}\,dt=\int_0^1 6t\sqrt{1+t^2}\,dt$$

let $u=1+t^2$
$du=2t\,dt$

$$=3\int_1^2\sqrt{u}\,du=2u^{(3/2)}\Big|_1^2=2(\sqrt{8}-1)$$

$$=\underline{2(2\sqrt{2}-1)}$$

9) $x=y^{(3/2)}$ $0<y<1$

$$L=\int_a^b\sqrt{\left(\frac{dx}{dy}\right)^2+1}\,dy \Rightarrow$$

$$=\int_0^1\sqrt{\left(\frac{3\sqrt{y}}{2}\right)^2+1}\,dy$$

let $u=1+\frac{9}{4}y$
$du=\frac{9}{4}dy$

$$=\int_1^{13/4}\sqrt{u}\;\frac{4}{9}du$$

$$=\frac{4}{9}\int_1^{13/4}\sqrt{u}\,du=\frac{4}{9}\cdot\frac{2}{3}\cdot u^{(3/2)}\Big|_1^{13/4}=\boxed{\frac{8}{27}\left(\left(\frac{13}{4}\right)^{(3/2)}-1\right)}$$

9-12, 14-16 find the exact lengths of the curves: Chapter 6.4 Skip Lester February 18, 2013

11) $y = \frac{x^2}{4} - \frac{1}{2}\ln x, 1 \leq x \leq 2$

$y' = (x/2) - (1/2x), 1 \leq x \leq 2$

$L = \int_1^2 \sqrt{1 + \left(\frac{x}{2} - \frac{1}{2x}\right)^2} dx$

$(x/2 - 1/2x)(x/2 - 1/2x)$

$= \int_1^2 \sqrt{1 + \left[\frac{x^2}{4} - \frac{1}{2} + \frac{1}{x^2}\right]} dx \quad x^2/4 - 1/4 - 1/4 + 1/4x^2$

$= \int_1^2 \sqrt{\frac{x^2}{4} + \frac{1}{2} + \frac{1}{4x^2}} = \int_1^2 \sqrt{\left(\frac{x}{2} + \frac{1}{2x}\right)^2} dx$

$L = \int_1^2 \left(\frac{x}{2} + \frac{1}{2x}\right) dx = \left.\frac{x^2}{4} + \frac{1}{2}\ln|x|\right]_1^2$

$= 1 + 1/2 \ln 2 - [1/4 - 0] = \underline{3/4 - 1/2 \ln 2}$

12) $x = a(\cos\theta + \theta\sin\theta) \quad y = a(\sin\theta - \theta\cos\theta)$

$0 \leq \theta \leq \pi \quad y = a\sin\theta - a\theta\cos\theta$

$x = a\cos\theta + a\theta\sin\theta$

$\frac{dx}{d\theta} = -a\sin\theta + [a\sin\theta + a\theta\cos\theta]$

$= a\theta\cos\theta$

$a\theta\sin\theta$
a let $f(\theta) = \theta$
let $g(\theta) = \sin\theta$
$\frac{d}{dx} f(\theta)g(\theta) = f'(\theta)g(\theta) + g'(\theta)f(\theta)$
$1 \cdot \sin\theta + \theta\cos\theta$

$\frac{dy}{d\theta} = a\cos\theta - [a\cos\theta - a\theta\sin\theta]$

$\frac{dy}{d\theta} = a\theta\sin\theta$

$L = \int_0^\pi \sqrt{(a\theta\cos\theta)^2 + (a\theta\sin\theta)^2} d\theta \quad \Rightarrow L = a\int_0^\pi \theta \, d\theta$

$L = a\int_0^\pi \sqrt{\theta^2\cos^2\theta + \theta^2\sin^2\theta} \, d\theta \quad = \boxed{\frac{a\pi^2}{2}}$

$L = a\int_0^\pi \theta \sqrt{\cos^2\theta + \sin^2\theta} \, d\theta$

10) $y = \sqrt{x - x^2} + \sin^{-1}(\sqrt{x})$

$y = (x - x^2)^{(1/2)} + \sin^{-1}(x^{(1/2)})$

$y' = \frac{1}{2\sqrt{x - x^2}} \cdot (1 - 2x) + \frac{d}{dx}\sin^{-1}(x^{(1/2)})$

$y' = \frac{1 - 2x}{2\sqrt{x - x^2}} + \left(\frac{1}{\sqrt{1 - x}} \cdot \frac{1}{2\sqrt{x}}\right)$

$y' = \frac{1 - 2x}{2\sqrt{x - x^2}} + \frac{1}{2\sqrt{x - x^2}}$

$y' = \frac{2 - 2x}{2\sqrt{x - x^2}} = \frac{1 - x}{\sqrt{x - x^2}}$

$\Longrightarrow L = \int_0^1 \sqrt{1 + \left(\frac{1 - x}{\sqrt{x - x^2}}\right)^2}$

$\Rightarrow L = \int_0^1 \sqrt{1 + \frac{1 - 2x + x^2}{x - x^2}}$

$\Rightarrow L = \int_0^1 \sqrt{\frac{x - \cancel{x^2} + \cancel{x^2} - 2x + 1}{x - x^2}} = \int_0^1 \sqrt{\frac{1 - x}{x(1 - x)}}$

$= \int_0^1 \sqrt{\frac{1}{x}} = \int_0^1 (x^{-1})^{(1/2)} = \int_0^1 x^{(-1/2)}$

$= \left[2x^{(1/2)}\right]_0^1 = \underline{2}$

14-16 Graph the curve | 6.4 | Integral calculus Shaun Lester February 18, 2013

14) $y = \frac{x^3}{3} + \frac{1}{4x} \quad 1 \leq x \leq 2$

$y' = x^2 + \left(\frac{-1}{4x^2}\right)$

$(y')^2 = x^4 - 1/2 + 1/16x^4$

$L = \int_1^2 \sqrt{1 + \left[\frac{16x^8 - 8x^4 + 1}{16x^4}\right]} dx$

$= \int_1^2 \sqrt{\frac{16x^8 + 8x^4 + 1}{16x^4}} dx$

$= \int_1^2 \sqrt{\frac{(4x^4+1)^2}{16x^4}} dx$

$= \int_1^2 \frac{4x^4+1}{4x^2} dx$

$= \int_1^2 \frac{dx}{4x^2} + \int_1^2 \frac{x^2}{1} dx$

$= \left.\frac{-1}{4x}\right]_1^2 + \left.\frac{x^3}{3}\right]_1^2$

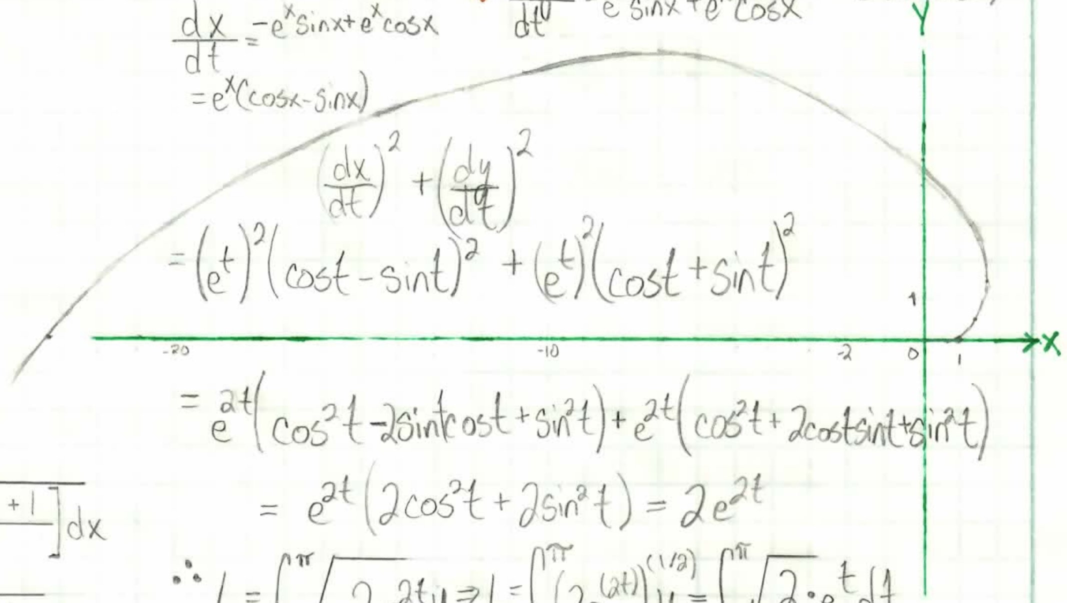
$= \frac{-1}{8} + \frac{1}{4} + \frac{7}{3}$

$= \frac{3}{24} + \frac{56}{24} = \frac{59}{24}$

15) $x = e^t \cos t \quad y = e^t \sin t, \ 0 \leq t \leq \pi$

$\frac{dy}{dt} = e^x \sin x + e^x \cos x = e^x(\sin x + \cos x)$

$\frac{dx}{dt} = -e^x \sin x + e^x \cos x$
$= e^x(\cos x - \sin x)$



$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$

$= (e^t)^2(\cos t - \sin t)^2 + (e^t)^2(\cos t + \sin t)^2$

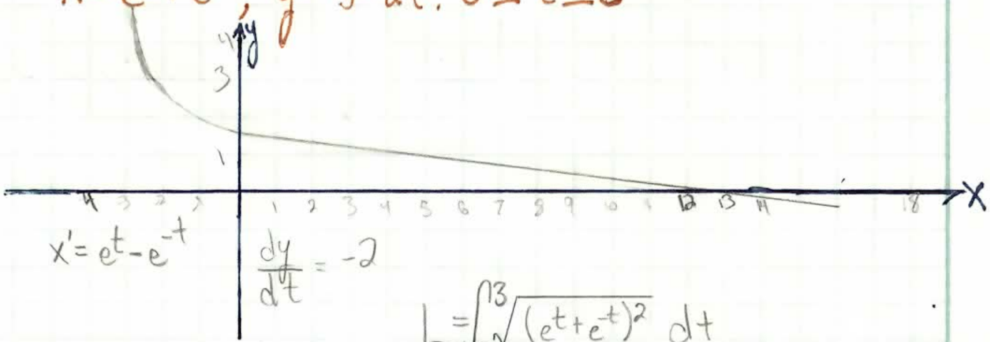
$= e^{2t}(\cos^2 t - 2\sin t \cos t + \sin^2 t) + e^{2t}(\cos^2 t + 2\cos t \sin t + \sin^2 t)$

$= e^{2t}(2\cos^2 t + 2\sin^2 t) = 2e^{2t}$

$\therefore L = \int_0^\pi \sqrt{2e^{2t}} dt \Rightarrow L = \int_0^\pi (2e^{(2t)})^{(1/2)} dt = \int_0^\pi \sqrt{2} \cdot e^t dt$

$= \sqrt{2} \int_0^\pi e^t dt = \sqrt{2} \cdot e^\pi - \sqrt{2} = \underline{\sqrt{2}(e^\pi - 1)}$

16) $x = e^t + e^{-t}, \ y = 5 - 2t, \ 0 \leq t \leq 3$



$x' = e^t - e^{-t}$

$\frac{dy}{dt} = -2$

$(e^t - e^{-t})^2 + (-2)^2$

$= e^{2t} + e^{-2t} - 2 + 4$

$= e^{2t} + e^{-2t} + 2$

$= (e^t + e^{-t})^2$

$L = \int_0^3 \sqrt{(e^t + e^{-t})^2} dt$

$= \int_0^3 e^t + e^{-t} dt$

$= \left[e^t - e^{-t}\right]_0^3$

$= e^3 - e^{-3} - [1 - 1]$

$= \underline{e^3 - e^{-3}} \approx 20$

;-5-7,9 Average Value of a Function Chapter 6.5 Skip Lester Calc II February 18, 2013

5/5 perfect

1) Find the average value

$f(x) = 4x-x^2, [0,4]$

$\frac{1}{b-a}\int_a^b f(x)dx$

$\frac{1}{4}\int_0^4 (4x-x^2)dx$

$= \frac{1}{4}\left[2x^2 - \frac{x^3}{3}\right]_0^4$

$= \frac{1}{4}\left[32 - \frac{64}{3}\right]$

$= 1/4\left[\frac{96-64}{3}\right]$

$= \frac{32}{12} = \boxed{\frac{8}{3}}$

5) $h(x) = \cos^4 x \sin x, [0,\pi]$

$(1/(b-a))\int_a^b f(x)dx$

$= 1/\pi\int_0^\pi \cos^4 x \sin x\, dx$ let $u = \cos x$
$du = -\sin x dx$

$= 1/\pi\int_{-1}^1 u^4 du = 1/\pi\left[u^5/5\right]_{-1}^1 = 1/\pi\cdot[2/5] = \underline{2/5\pi}$

6) $h(u) = (3-2u)^{-1}, [-1,1]$

$\frac{1}{b-a}\int_a^b f(x)dx$

$= \frac{1}{2}\int_{-1}^1 \frac{1}{3-2u}du$ let $\omega = 3-2u$
$d\omega = -2du$

$= \frac{1}{2}\int_5^1 \frac{1}{\omega}\frac{d\omega}{-2} = \frac{1}{4}\int_1^5 \frac{1}{\omega}d\omega = \frac{1}{4}\left[\ln\omega\right]_1^5$

$= \underline{\frac{\ln 5}{4}}$

7) find f_{avg}. find c such that $f(c) = f_{avg}$. sketch f + its average rectangle of equal area.

$f(x) = (x-3)^2, [2,5]$

$1/(b-a)\int_a^b f(x)dx$

$= 1/3\int_2^5 (x-3)^2 dx$ let $u = x-3$
$du = dx$

$= 1/3\int_{-1}^2 u^2 du = 1/3\left[u^3/3\right]_{-1}^2$

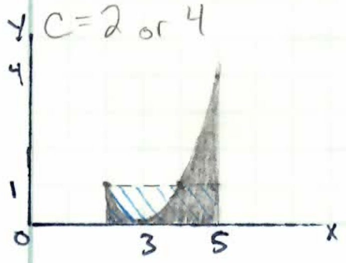
$= 8/9 + 1/9 = 1$

$(x-3)^2 = 1$

$x-3 = 1 \quad x-3 = -1$

$x = 4 \quad x = 2$

$c = 2$ or 4



9) $f(x) = 2\sin x - \sin 2x, [0,\pi]$

$1/(b-a)\int_a^b f(x)dx = 1/\pi\int_0^\pi 2\sin x - \sin 2x$

$= 2/\pi\int_0^\pi \sin x\, dx - 1/\pi\int_0^\pi \sin 2x dx$ let $u = 2x$
$du = 2dx$

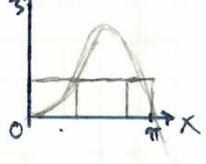
$= \frac{2}{\pi}\left[-\cos x\right]_0^\pi - \frac{1}{2\pi}\int_0^{2\pi} \sin u\, du$

$= \frac{2}{\pi}[1+1] - \frac{1}{2\pi}\left[-\cos u\right]_0^{2\pi}$

$= \frac{4}{\pi} - \frac{1}{2\pi}[-1+1] = \frac{4}{\pi}$

$2\sin c - \sin 2c = 4/\pi$

$c \approx 1.238$ or 2.808



Average Value of functions | Chapter 6.5 | Skip Lester February 18, 2013

12) find the numbers b such that the average value of $f(x) = 2+6x-3x^2$ on the interval $[0,b]$ is equal to 3.

$$\frac{1}{b-a}\int_a^b f(x)\,dx \rightarrow \frac{1}{b}\int_0^b 2+6x-3x^2 = \frac{1}{b}\left[2x+3x^2-x^3\right]_0^b$$

$$= \frac{1}{b}\left[2b+3b^2-b^3\right] = -b^2+3b+2 \Rightarrow -b^2+3b+\underset{-3}{2} = \underset{-3}{3} \Rightarrow -b^2+3b-1=0$$

$$a=-1 \quad b=+3 \quad c=-1 \Rightarrow \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-3\pm\sqrt{9-4}}{-2} = \underline{\frac{3+\sqrt{5}}{2} \text{ or } \frac{3-\sqrt{5}}{2}}$$

13) The table gives values of a continuous function.
Use Simpson's Rule to estimate the average value of f on $[20, 50]$

$\Delta X = 30$

X	20	25	30	35	40	45	50
$f(x)$	42	38	31	29	35	48	60

$$\frac{1}{b-a}\int_a^b f(x)\,dx \approx \frac{1}{50-20}\cdot S_6$$

$$= \frac{1}{\cancel{30}}\cdot\frac{\cancel{30}}{3\cdot 6}\left[42+(4\cdot 38)+(2\cdot 31)+(4\cdot 29)+(2\cdot 35)+(4\cdot 48)+60\right]$$

$$= \frac{1}{18}(694) = \underline{\frac{347}{9}} \approx \underline{38.6}$$

15) In a certain city the temperature (in °F) t hours after 9:00am is modeled by the function $T(t) = 50+14\sin(\pi t/12)$
Find the average from 9:00 an to 9:00pm $\Delta t = 12$

$$1/12\int_0^{12}\left[50+14\sin(\pi t/12)\right]dt \Rightarrow 1/12\left[50t - 14\cdot 12/\pi \cos\frac{\pi t}{12}\right]_0^{12}$$

$$= \frac{1}{12}\left[(50\cdot 12) - \left[14\left(\frac{12}{\pi}\right)(-1)\right] - \left[-14\cdot\frac{12}{\pi}(1)\right]\right] = 50+\frac{28}{\pi}\text{°F} \approx 59\text{°F}$$

17) The linear density of a rod 8m long is $12/\sqrt{x+1}$ kg/m, where x is measured in meters from one end of the rod. Find the average density of said rod.

$$\frac{1}{b-a}\int_a^b f(x)\,dx \Rightarrow 1/8\int_0^8 \frac{12}{\sqrt{x+1}}\,dx \rightarrow 3/2\int_0^8 \frac{1}{\sqrt{x+1}} \Rightarrow \left(\frac{3}{2}\cdot 2\cdot\sqrt{x+1}\right)\Big|_0^8 = 3\sqrt{9}-3\sqrt{1} = \underline{6 \text{ kg/m}}$$

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18) If a freely falling body starts from rest, then its displacement is given by $s = \frac{1}{2} g t^2$ Let the velocity after a time T be v_T. Show that if we compute the average of the velocities with respect to t we get $v_{avg} = \frac{1}{2} v_T$, but if we compute the average with respect to s we get $\frac{2}{3} V_T$

$S = \frac{1}{2} g t^2$ $V_T = \int S(t)\,dt = \int \frac{1}{2} g t^2\,dt = g/2 \int t^2\,dt$ $g/2 \cdot \frac{t^3}{3} = g t^3/6$

$\frac{1}{T}\int_0^T \frac{1}{2} g t^2\,dt \rightarrow \frac{g}{2T}\int_0^T t^2\,dt = \frac{g}{2T} \cdot T^3 = \frac{g T^2}{2}$

Solowan's in-class solution

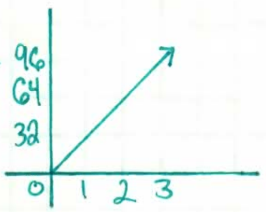
V_T = velocity after time 'T' $S = \frac{1}{2} V_T$

$V_{Ave} = \frac{1}{2} V_T$ w.r.t time $\cancel{V_T = \int_0^t}$ $S = \frac{1}{2} g t^2$

V_{Ave} $\frac{2}{3} V_T$ w.r.t S $S' = v = 32 ft/s \cdot t$

$= 32t$

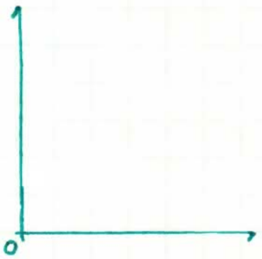
T	V
0	0
1	32
2	64
3	96
T	V_T



$\rightarrow V(t) \frac{1}{2} V_T = V(t)$

$V(1) = 16 ft/s$

S	V
16	32
32	64



I tried!

5/5

Taylor Polynomials section 1 Skip Lester February 24, 2013

1. Let $T_2(x) = 1 + x + \frac{x^2}{2}$ be the taylor polynomial of degree 2 for $f(x) = e^x$, centered at $a=0$. Verify directly by taking their derivatives that $T_2(x)$ and $f(x)$ satisfy the three conditions $T_2(0) = f(0)$, $T_2'(0) = f'(0)$, $T''(0) = f''(0)$

① $T_2(0) = 1 + 0 + 0/2 = 1 \quad f(0) = e^0 = 1$ ✓

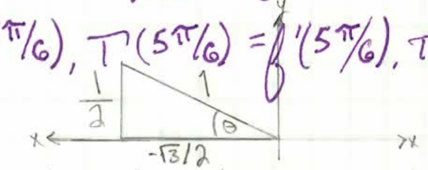
② $T_2'(x) = 0 + 1 + x = x+1 \quad f'(x) = e^x$

$T_2'(0) = 0 + 1 = 1 \quad f'(0) = e^0 = 1$ ✓

③ $T_2''(x) = 1 \quad T(0) = 1 \quad f''(x) = e^x \quad f''(0) = e^0 = 1$ ✓ ✓

---

2. Let $T_3(x) = 1/2 - \frac{\sqrt{3}}{2}\left(x - \frac{5\pi}{6}\right) - \frac{1}{4}\left(x - \frac{5\pi}{6}\right)^2 + \frac{\sqrt{3}}{12}\left(x - \frac{5\pi}{6}\right)^3$ be the taylor polynomial of degree ~~two~~ three for $f(x) = \sin x$, centered at $x = 5\pi/6$. Verify directly by taking their derivatives that $T_3(x)$ and $f(x)$ satisfy the four conditions: $T_3(5\pi/6) = f(5\pi/6)$, $T'(5\pi/6) = f'(5\pi/6)$, $T''(5\pi/6) = f''(5\pi/6)$, $T_3^{(3)}(5\pi/6) = f^{(3)}(5\pi/6)$



$x - \frac{5\pi}{6}\left(x - \frac{5\pi}{6}\right)$

$\frac{d}{dx}\left(x^2 - \frac{5\pi}{3}x + \frac{25\pi}{36}\right)$

$= 2x - \frac{5\pi}{3}$ / 4

$= -\frac{1}{2}x + \frac{5\pi}{12}$

$\frac{d}{dx}\frac{\sqrt{3}}{12}\left(x - \frac{5\pi}{6}\right)^3$

$= \frac{\sqrt{3}}{4}\left(x - \frac{5\pi}{6}\right)^2$

$= \frac{\sqrt{3}}{4}x^2 - \frac{5\pi x}{4\sqrt{3}} + \frac{25\pi^2}{18\sqrt{3}}$

$T_3(5\pi/6) = 1/2 - \sqrt{3}/2(0) - 1/4(0) + \sqrt{3}/12(0) = 1/2 \quad f(5\pi/6) = \sin(5\pi/6) = 1/2$ ✓

$T_3'(x) = -\frac{\sqrt{3}}{2} + \frac{1}{2}\left(-x + \frac{5\pi}{6}\right) + \frac{\sqrt{3}}{4}\left(x - \cancel{\frac{5\pi}{6}}\right)^2 \quad f'(5\pi/6) = \cos 5\pi/6 = -\frac{\sqrt{3}}{2}$ ✓

$T_3'(5\pi/6) = \frac{-\sqrt{3}}{2}$

$f''(5\pi/6) = -\sin 5\pi/6 = -\frac{1}{2}$ ✓

$f^{(3)}(5\pi/6) = -\cos 5\pi/6 = \frac{\sqrt{3}}{2}$ ✓

$T_3''(x) = -1/2 + \sqrt{3}/2(x - 5\pi/6)$

$T_3''(5\pi/6) = -1/2$

$T_3^{(3)}(x) = \frac{\sqrt{3}}{2}$

$T_3^{(3)}(5\pi/6) = \sqrt{3}/2$

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#3.) Let $T_3(x)=F(a)+F'(a)(x-a)+\frac{F''(a)(x-a)^2}{2}+\frac{F'''(a)(x-a)^3}{6}$ be the Taylor polynomial of degree 3 of the function $F(x)$, centered at $x=a$.

a) Find $T_3'(x)$, $T_3''(x)$, and $T_3'''(x)$

$$T_3'(x)=F'(a)+F''(a)(x-a)+\frac{F'''(a)(x-a)^2}{2}$$

$$T_3''(x)=F''(a)+F'''(a)(x-a)$$

$$T_3^{(3)}(x)=F^{(3)}(a)$$

b) evaluate at $x=a$ these derivatives

$$T_3'(a)=F'(a)$$

$$T_3''(a)=F''(a)$$

$$T_3^{(3)}(a)=F^{(3)}(a)$$

#4) The function $f(x)$ is approximated near $x=0$ by the Taylor $T_2(x)=5-7x+8x^2$

Find the value of $f'(0)$, $f''(0)$, + $f(0)$

$T_2(x)=8x^2-7x+5$ $T(0)=5 \therefore f(0)=5$

$T_2'(x)=16x-7$, $T'(0)=-7 \therefore f'(0)=-7$

$T_2''(x)=16$ $T''(0)=16 \therefore f''(0)=16$ ✓

5) Suppose g is a function which has continuous derivatives, and suppose also that $g(0)=3$, $g'(0)=2$, $g''(0)=1$, and $g'''(0)=-3$.

a). What is T_2, center at 0. b) T_3, center at 0. C) Use $T_2(x)+T_3(x)$ to approx $g(.1)$

a) $T_2=3+2(x-0)+\frac{1(x-0)^2}{2}$

$$=\frac{x^2}{2}+2x+3$$

b)

$$T_3=3+2x+\frac{x^2}{2}+\frac{-3(x-0)^3}{6}$$

$$=\frac{-x^3}{6}+\frac{x^2}{2}+2x+3$$

$g(.1)\approx 3.2$

6) For the function $f(x)=\ln(x)$

a) list the first four derivatives of $f(x)$

$f(x)=\ln(x)$ $f''(x)=-1/x^2$ $f^{(4)}x=-6/x^4$

$f'(x)=1/x$ $f'''(x)=2/x^3$

b) What are the values evaluated at $a=1$?

$f'(1)=1$; $f''(1)=-1$, $f'''(1)=2$, $f^{(4)}(1)=-6$

c) write T_4 in "long form"

$$T_4(x)=\ln(a)+\frac{1}{a}(x-a)-\frac{1}{a^2}\cdot\frac{1}{2}(x-a)^2+\frac{1}{3a^3}(x-a)^3-\frac{1}{4a^4}(x-a)^4$$

. awesome.

d) Summation notation $T_4(x)=\ln(a)+\sum_{i=1}^{4}(ia^i)^{-1}(x-a)^i$

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7.) For the function $f(x) = 2+3x+4x^2+5x^3$ $f'(x) = 3+8x+15x^2$ $f''(x) = 8+30x$ $f'''(x) = 30$ $f^{(4)}(x) = 0$

a) find $T_{0-5}(x)$, centered at $a=0$ $T_0(x) = 2$ $T_2(x) = 2+3x+\frac{8(x)^2}{2} = 4x^2+3x+2$

b) what is the signifigance of $T_0(x)$? $T_1(x) = 2+3(x)$

its, the function evaluated at a

c) what is $T_{103}(X)$? $T_3(x) = 2+3x+4x^2+\frac{30(x)^3}{6} = 5x^3+4x^2+3x+2 = f(x)$

$T_{103}(x) = 5x^3+4x^2+3x+2 = f(x)$

8) For the function $f(x) = \ln(x)$: $\frac{d}{dx}\ln(x) = \frac{1}{x}$

a) find an equation of the line tangent to $f(x)$ at $x=1$. $f'(1)=1$ $f(1)=0$

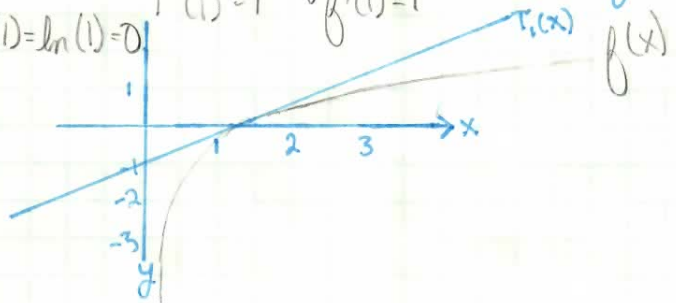
$0 = 1x+B$ $B=-1$ $\underline{y = x-1}$

b) find a function $T_1(x)$ that satisfies $T_1'(1) = f'(1)$ and ~~$T_1(x) = f(x)$~~ [illegible]

$\underline{T_1(x) = x-1}$ $T'(x) = 1$ $T'(1) = 1$ $f'(x) = 1/x$ $f'(1) = 1$ $T_1(1) = f(1)$

$T(1) = 0$, $f(1) = \ln(1) = 0$

c) graph it



9) For the function $f(x) = \ln(x)$: $1/x$ $-1/x^2$

$(x-1)(x-1)$
x^2-2x+1

a) Find $T_2(x)$ such that $T_2(1) = f(1)$, $T_2'(1) = f'(1)$, $T_2''(1) = f''(1)$.

$T_2(x) = f(1) + f'(1)(x-1) + \frac{f''(1)(x-1)^2}{2}$

$T_2(x) = \ln(1) + 1/1(x-1) + \frac{(-1(x-1)^2)}{2}$

$T_2(x) = 0 + (x-1) - 1/2(x-1)^2$

$T_2(x) = \frac{2x-2}{2} - \left(\frac{x^2-2x+1}{2}\right)$ $\boxed{\frac{-(x^2)+4x-3}{2}}$

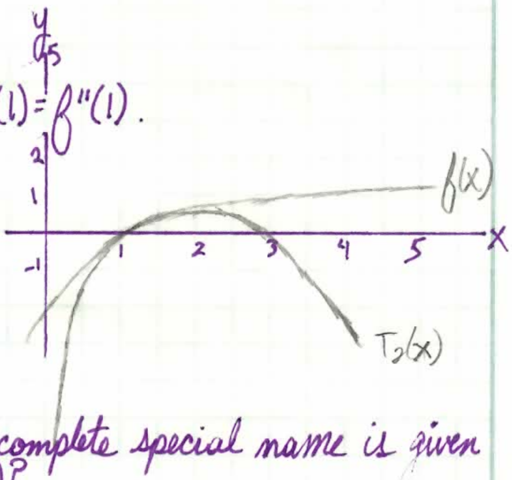
$T_2(x) = \frac{-(x^2)+4x-3}{2}$ $T_2'(x) = -x+2$ $T_2'(1) = -1+2 = 1$

$T_2(1) = -1^2+4-3 \cdot 1/2 = 0$ $f'(1) = 1/1 = 1$

$T_2''(x) = -1$

$f(1) = \ln(1) = 0$ $f''(1) = -1/1^2 = -1$

b) graph em



c) What complete special name is given to $T_2(x)$?

Taylor polynomial of degree two centered at $x=1$ for $\ln(x)$.

d) $f(1)$, $T_2(1)$, $f(2)$, $T_2(2)$, $f(0)$, $T_2(0)$

$f(1)=0$, $T_2(1)=0$. $f(2) = \ln 2$ $f(0) = -\infty$

$T_2(2) = 1/2$ $T_2(0) = -3/2$

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#10.) find $T_4(x)$ for $\ln(x)$, centered at $a=2$ $\quad f(a)+f'(a)(x-a)+\frac{f''(a)(x-a)^2}{2!}+\frac{f^3(a)(x-a)^3}{3!}+\frac{f^4(x-a)^4}{4!}$

$$\overset{f(x)}{\ln(x)} \rightarrow \overset{f'(x)}{1/x} \rightarrow \overset{f''(x)}{-1/x^2} \rightarrow \overset{f^{(3)}(x)}{2/x^3} \rightarrow \overset{f^{(4)}(x)}{-6/x^4}$$

$$T_4(x) = \ln 2 + \tfrac{1}{2}(x-2) + \tfrac{1}{4}\cdot\tfrac{1}{2}(x-2)^2 + \left[\tfrac{2}{8}\cdot\tfrac{1}{6}(x-2)^3\right] + \left[-\tfrac{6}{16}\cdot\tfrac{1}{24}(x-2)^4\right]$$

$$= \ln 2 + \tfrac{1}{2}(x-2) + \tfrac{1}{8}(x-2)^2 + \tfrac{1}{24}(x-2)^3 - \tfrac{1}{64}(x-2)^4$$

b $\quad T_4(2.2) = \ln 2 + \tfrac{1}{2}(.2) + \tfrac{1}{8}(.2)^2 + \tfrac{1}{24}(.2)^3 - \tfrac{1}{64}(.2)^4 = 0.798456$

$\ln(2.2) = 0.788457 \;\|\; T_4(2.2) - f(2.2) = .00999815 \approx 1/100$

11) Find the parabola that best fits the unit circle $x^2+y^2=1$ at $(0,1)$

$$y = \pm\sqrt{1-x^2} = (1-x^2)^{1/2} \Rightarrow y' = (1-x^2)^{-1/2}\cdot -2x\cdot 1/2 = \frac{-x}{\sqrt{1-x^2}} = (-x)(1-x^2)^{-1/2} = f'(x)$$

$$f''(x) = \frac{d}{dx}\left[-x(1-x^2)^{-1/2}\right] \Rightarrow \text{let } f(x)=-x,\ g(x)=(1-x)^{-1/2}$$

$$f'(x)=-1 \qquad g'(x) = (-1/2)\cdot(1-x^2)^{-3/2}(-2x) = \frac{x}{(1-x^2)^{3/2}}$$

$$[f'(x)\cdot g(x)] + [g'(x)\cdot f(x)] \Rightarrow f''(x) = \left[-1(1-x^2)^{-1/2}\right] + \left[\frac{-x^2}{(1-x^2)^{3/2}}\right] = \frac{-x^2}{(1-x^2)^{3/2}} - \frac{1}{(1-x^2)^{1/2}}$$

$T_2(x)$ is the parabola we seek, centered at $a=0$

$$= \frac{-x^2}{(1-x^2)^{3/2}} - \frac{(1-x^2)}{(1-x)^{3/2}} = \frac{-1}{(1-x)^{3/2}}$$

To recap, $F(x) = \sqrt{1-x^2}$

$f'(x) = \dfrac{-x}{\sqrt{1-x^2}}$

$f''(x) = \dfrac{-1}{\sqrt{(1-x^2)^3}}$

$$T_2(x) = f(a) + f'(a)(x-a) + f''(a)(x-a)^2(1/2)$$

$$= \sqrt{1} + 0 + \left[\left(\frac{-1}{1^{3/2}}\right)(x-0)^2\cdot\frac{1}{2}\right]$$

$$= \underline{\frac{-x^2}{2} + 1}$$

b) use this to estimate the y value of the unit circle when $x=0.1$

$$T_2(0.1) = \frac{-(.1)^2}{2} + 1 = \underline{.995}$$

12) Find the taylor polynomial of degree 4, centered at $a=0$, for the function $f(x) = e^{x^2}$ Using mathematica, $f'(x) = 2xe^{x^2}$; $f''(x) = 4x^2e^{x^2} + 2e^{x^2}$; $f'''(x) = 8x^3e^{x^2} + 12e^{x^2}$

$f^{(4)}(x) = 16x^4e^{x^2} + 48x^2e^{x^2} + 12e^{x^2}$ and $T_4(x) = \left[f(a) + f'(a)(x-a) + \tfrac{1}{2}(f''(a)(x-a)^2) + \left(\tfrac{1}{6}(f'''(a)(x-a)^3\right) + \frac{f^{(4)}(x-a)^4}{24}\right]$

$$\therefore T_4(x) = 1 + 0 + \left[\frac{1}{2}\cdot 2(x-0)^2\right] + \left[\frac{1}{6}\cdot 12(x)^3\right] + \left[\frac{1}{24}\cdot 12(x)^4\right]$$

$$= \underline{1 + x^2 + \cancel{2x^3} + \frac{x^4}{2}}$$

is this a typo?

b) Compare this w/ $T_2(x)$ for e^x, $a=0$

$$f(a) + f'(a)(x-a) + f''(a)(x-a)^2\cdot\tfrac{1}{2}$$

$$= \underline{1 + x + \frac{x^2}{2}}$$ it's similar?

$$T_2(x) = 1 + 0 + 2(x^2) \div 2$$

$$= x^2 + 1$$

#12

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12) c)

$$T_{10}(x) = 1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6} + \frac{x^8}{24} + \frac{x^{10}}{120}$$

$$T_{10}(x) = \frac{x^0}{0!} + \frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{24} + \frac{x^{10}}{120}$$

$$T_n(x) = \sum_{n=0}^{n} \frac{x^{2n}}{n!}$$

d) $F(x) = e^{-2x}$

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13) (a) $T_3(x)$ for $F(x)=\ln(x)$ $\frac{1}{x}$ $\frac{-1}{x^2}$ $\frac{2}{x^3}$

centered at $a=2$

$$\sum_{K=0}^{3} \frac{F^{(K)}(2)}{K!}(x-2)^K$$

$$= \ln 2 + (1/2(x-2)) - (1/8(x-2)^2) + (1/24(x-a)^3)$$

(b) T_3 for $f(x)=\sqrt{1+x}$ centered at $a=0$.

$$f'(x)=\frac{1}{2\sqrt{1+x}} \quad f''(x)=\frac{-1}{4(1+x)^{3/2}} \quad f^{(3)}(x)=\frac{3}{8(1+x)^{5/2}}$$

$$= 1 + 1/2x - \left[1/4(x^2)\cdot\frac{1}{2}\right] + \left[\frac{3}{8}\cdot\frac{1}{6}\cdot x^3\right]$$

$$= \frac{1}{16}x^3 + \frac{x^2}{8} + \frac{x}{2} + 1$$

c) $T_3(x)$ for $\sin(x)$, $a=\pi/3$.

$\cos x$ $-\sin x$ $-\cos x$

$$f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 + \frac{1}{6}f'''(a)(x-a)^3$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2}\left(x-\frac{\pi}{3}\right) + \left[\frac{1}{2}\cdot\frac{-\sqrt{3}}{2}\left(x-\frac{\pi}{3}\right)^2\right] + \left[\frac{1}{6}\cdot\frac{-1}{2}\left(x-\frac{\pi}{3}\right)^3\right]$$

$$= \frac{\sqrt{3}}{2} + \frac{x-\pi/3}{2} - \frac{\sqrt{3}}{4}\left(x-\frac{\pi}{3}\right)^2 - \frac{1}{12}\left(x-\frac{\pi}{3}\right)^3\Big]$$

d) $f(x)=\cos x$, $T_4(x)$, $a=\pi$

$$\underset{f(x)}{\cos(x)} \to \underset{f'(x)}{-\sin x} \to \underset{f''(x)}{-\cos(x)} \to \underset{f'''(x)}{\sin x} \to \underset{f^{(4)}(x)}{\cos(x)}$$

$$f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \frac{f^{(4)}(x-a)^4}{4!}$$

$$-1 + \frac{(x-\pi)^2}{2} - \frac{(x-\pi)^4}{24}$$

e) $f(x)=\frac{1}{2-x}$ degree n, $a=0$ $\displaystyle\sum_{K=0}^{n} \frac{1}{2^{K+1}}\cdot x^K$

14) Show how you can use the taylor polynomial of degree 3 centered at $a=0$, which says that $\sin x \approx x - \frac{x^3}{3!}$, to explain why $\lim_{x\to 0}\frac{\sin(x)}{x}=1$

divide by x

is this l'hopitals?

$$T_3(x) = f(a) + f'(a)(x-a) + \left[f''(a)(x-a)^2\cdot\frac{1}{2}\right] + \left[f^{(3)}(a)(x-a)^3\cdot 1/6\right]$$

$\sin x$ $f(x)$ $f(0)=0$

$\cos x$ $f'(x)$ $f'(0)=1$

$-\sin x$ $f''(x)$ $f''(0)=0$

$-\cos x$ $f'''(x)$ $f'''(0)=-1$

$$1(x) - \frac{(x)^3}{6} \approx \sin(x) \therefore \frac{\sin(x)}{x} = \frac{x-\frac{x^3}{6}}{x} = 1 - \frac{x^2}{6} \quad 1 - 0/6 = 1 \square$$

Taylor Section 2

Skip Lester February 25 2013

1) a) Determine a better value for M_2 in Example 2.1

$$|\sin x - x| = |f(x) - T_1(x)| = |R_1(x)| \leq \frac{M_2}{2!}|x-a|^2$$

For M, we take a number slightly higher than the maximum value $(n+1)^{th}$ derivative. The $(n+1)^{th}$ derivative in this case refers to the second derivative of sin(x). The second derivative of $\sin(x) = f''(x) = -\sin x$ so the absolute maxima of $|-\sin x|$ on $[0, 0.1]$ is at $x = .1$, $y = 0.0998334...$

I take for M_2 $\underline{.1} \Rightarrow \frac{.1}{2!}|.1-0|^2 = .05|.1|^2 = .05(.01) = \overset{b)}{\underline{.0005}}$

2) For the $T_4(x)$ of $\ln(x)$ centered at $a=2$:

a) The value of n should be 4.

b) what is the $(n+1)^{st}$ derivative in this case? The fifth derivative of $\ln(x)$ $f^{(5)}(x) = \frac{24}{x^5}$ c) decreasing d) Over what interval must the graph of the $(n+1)^{st}$ derivative be examined in order to ascertain a value for M? from $a=2$ to $x=2.2$ e) Do you expect to find the best value for $M_{(n+1)}$ on the left, right, or center? $24/x^5$ is decreasing; the <u>left.</u>

3) Let $f(x) = \sqrt{1+x}$ a) find $|f^{(4)}(x)|$ and show that the function $g(x) = |f^{(4)}(x)|$ is decreasing for $x \geq 0$. Conclude $\max_{x\geq 0}|f^{(4)}(x)| = |f^{(4)}(0)|$

$f(x) = \sqrt{1+x}$ $f''(x) = \frac{-1}{4(x+1)^{3/2}}$ $f^{(4)}(x) = \frac{-15}{16}(x+1)^{-7/2}$

$f'(x) = \frac{1}{2\sqrt{1+x}}$ $f'''(x) = 3/8(x+1)^{-5/2}$ $g(x) = |f^{(4)}(x)| = 15/16(x+1)^{-7/2}$

inversely proportional so the function is decreasing.

b) give an upper bound for the error $f(x) - T_3(x)$, $a=0$, $0 \leq x \leq .1$

$$|R_3(x)| \leq \frac{M_4}{4!}|x-0|^4 = \frac{(15/16)}{24}(.1)^4 = 0.000004$$

c) $T_3(x) = 1 + \frac{x}{2} - x^2/8 + x^3/16$

$f(x) - T_3(x) \rightarrow f(.1) - T_3(.1) = .0000003652 \leq .000004$

4) For the taylor polynomial of degree 4 centered at $a=0$ for $f(x)=\sin x$

a) According to TAylor's inequality, what is the largest possible difference between the taylor polynomial and the function when estimating $f(0.75) \leq \frac{M_5}{5!}|.75|^5$

F $f(x)=\sin x$

1 $f'(x)=\cos x$

2 $f''(x)=-\sin x$

3 $f'''(x)=-\cos x$

4 $f^{4}(x)=\sin x$

$f^{(5)}(x)=\cos x$

$f^{(5)}(x)=\cos x$ is decreasing on $[0,.75]$, ∴ max at $\cos(0)=1$

$|R_n| \leq \frac{1}{5!}|.75^5| = \frac{1}{120}(.75^5) = \underline{.00198}$

What would you expect $|R_4(0.75)|$ to be larger are smaller, or the same if we centered our taylor at $\pi/4$? Smaller ∴ $(x-a)$ is less

c) $\frac{1}{120}\left|.75-\pi/4\right|^5 = \underline{4.7\times10^{-10}}$

5) For $g(x)=e^{2x}$, (a) What is $T_2(x), a=0 = f(a)+f'(a)(x-a)+[f''(a)(x-a)^2\cdot 1/2!]$

$e^0=1$ $g'(x)=e^{2x}\cdot 2 = 2e^{2x}$ $g''(x)=2e^{2x}\cdot 2 = 4e^{2x}$ $g'''(x)=8e^{2x}$ $g^{(4)}(x)=16e^{2x}$ $T_2(x)=1+2(x)+\frac{4(x^2)}{2!}$

(b) What is the maximum error we would expect in using $T_2(x)$ to estimate $g(.1)$? $\underline{T_2(x)=2x^2+2x+1}$

maximum of $g'''(x)=8e^{2x}$ on $[0,0.1]$ $g'''(x)$ is increasing ∴ max @ $x=.1$ $=9.77122$

$|g(.1)-T_2(.1)| \leq \frac{M_3}{3!}|x-a|^3$

$|R_2(.1)| \leq \frac{9.77}{6}|.1^3| \leq \underline{0.00162833}$

c) Is the error estimate larger, smaller, or the same as example 2.2's error estimate for $T_2(0.1)$, centered at $a=0$, for $f(x)=e^x$? What is it about the graphs that accounts for this $R_2(x)=0.000184195$, which is less. e^{2x} grows faster

d) $e^{.1}-[1+.1+\frac{.1^2}{2}]=0.000170918$

e) $e^{2(.1)}-[1+2(.1)+2(.1)^2]=0.00140276$

6) How many terms are necessary to guarantee the taylor polynomial, centered at $a=0$, is accurate to within 0.05 of the exact value when $x=0.3$ for:

(a) $f(x)=e^x$ $f'(x)=e^x$ $f^{(n)}(x)=e^x$ (increasing) $M=e^{.3}$

$|R_n(0.3)| \leq \frac{M_{n+1}}{(n+1)!}|0.3-0|^{n+1} \leq 0.05$ $\frac{e^{.3}}{2}|.3|^2 = 0.06 \nleq 0.05$

$\therefore$ $n=1$ is a bust

$\frac{e^{.3}}{3!}|.3|^3 = 0.006 < 0.05 \therefore 2$ terms

$n=2$

(b) $f(x)=\sin(x) \Rightarrow \cos(x) \Rightarrow -\sin(x) \Rightarrow -\cos(x)$

$|R_n(0.3)| \leq \frac{M_{n+1}}{(n+1)!}|.3|^{n+1} \leq .05 \rightarrow \frac{\cos(0)}{2}|.3|^2 = 0.0132 \leq 0.05$

$\not>$ cos is decreasing on $[0, 0.3]$

$n=1 \therefore$ 1 term is sufficient

(c) $f(x)=\cos(x) \rightarrow -\sin(x) \rightarrow -\cos(x) \rightarrow \sin(x)$ $-\sin(x)$ decreasing $\therefore$ max on $[0, 0.3]$ @ x

$|R_n(0.3)| \leq \frac{M_{n+1}}{(n+1)!}|.3|^{n+1} \leq 0.05$ $\frac{1}{1+1}(.3)^{1+1} = 0.0132 \leq .05$

one term

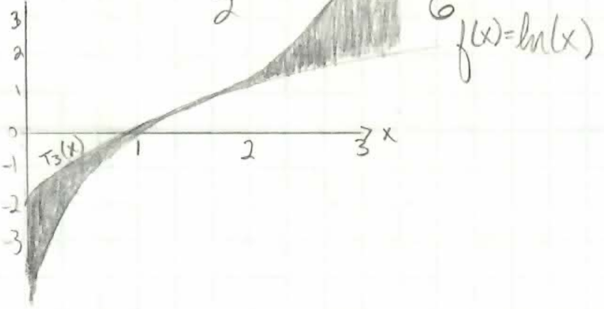
7) Draw a graph of $T_3(x)$, $a=1$, $f(x)=\ln(x)$, illustrating the exact error in using $T_3(x)$ to estimate $\ln(x)$

$\ln(x) \Rightarrow 1/x \Rightarrow -x^{-2} \Rightarrow 2x^{-3} \rightarrow -6x^{-4} \rightarrow 24x^{-5}$

$T_3(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2} + \frac{f^{(3)}(a)(x-a)^3}{6}$

$T_3(x) = 0 + (x-1) + \frac{-1(x-1)^2}{2} + \frac{2(x-1)^3}{6}$

$= \frac{(x-1)^3}{3} - \frac{(x-1)^2}{2} + x - 1$



b) $|R_3(x)|$ is an area, changing w/ x

I suggest using integration

8) Let $T_1(x)$ + $T_2(x)$ be taylor polynomials of degree one and two for $f(x)=\sin x$ @ $a=0$.

$\sin(0)=0$
$\cos(0)=1$

Show $T_1(x) = T_2(x)$ $T_1(x) = f(a) + f'(a)(x-a)$ $T_2(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!}$

$f(x) = \sin x$ $T_1(x) = 0 + 1(x) = x$ $T_2(x) = 0 + 1(x-a) + 0 = x$

$f'(x) = \cos x$

(b) Use Taylor's inequality with $n=2$ to get a better upper bound on $|\sin x - x|$ for $|x| \leq 0.1$ $2+1=3$

$f''(x) = -\sin x$ $f''(0)=0$

$R_2 \leq \frac{M_3}{3!}|x-0|^3 = \frac{1}{6}|.1^3| = 1.\bar{6} \cdot 10^{-4}$

For $f(x) = \sin x,\ a = 0$ Taylor HW Section 2 Skip Lester

(a) 9) show that for n, odd,

$$T_n(x) = T_{n+1}(x)$$

$f(x)$ $\sin x$ $f(a) = 0$

$f'(x)$ $\cos x$ $f'(a) = 1$

$f''(x)$ $-\sin x$ $f''(a) = 0$

$f^{(3)}(x)$ $-\cos x$ $f^{(3)}(a) = -1$

$f^{(4)}(x)$ $\sin x$ $f^{(4)}(a) = 0$

$$T_3(x) = \frac{f(a)}{0!} + \frac{F'(a)(x-a)}{1!} + \frac{f''(a)(x-a)}{2!} + \frac{f^{3}(a)(x-a)^3}{3!}$$

$$T_3(x) = 0 + x + 0 + \left[\frac{-1(x^3)}{6}\right] = T_4(x) = 0 + x + 0 - \frac{x^3}{6} + 0$$

$$= x - \frac{x^3}{6}$$

(b) When n is odd, find an upper bound on $|\sin(x) - T_n(x)|$ for $|x| \leq 0.1$

$$|R_n(0.1)| \leq \frac{M_{n+1}}{\quad}$$

exercises 5/5 Taylor Section 3 Skip Lester February 26, 2013

1) Consider the infinite series $\sum_{K=0}^{\infty}(-1)^K$. Write down the values for the first four partial sums for this series. Find the sum of this series if it exists. $S_0 = a_0$ $S_1 = a_0 + a_1$, $S_2 = a_0 + a_1 + a_2$

$S_0 = -1^0 = 1$ $S_1 = 1 + (-1^1) = 0$ $S_2 = 0 + (-1^2) = 1$ $S_3 = 1 + (-1^3) = 0$

This series oscillates, so $\lim_{K \to \infty} S_K$ DNE.

2) The series $\sum_{K=0}^{\infty} \frac{1}{2^K}$ is a geometric series. What is its sum?

$\frac{a}{1-r}$ $ar^K \to \frac{1}{2^0} = 1 \therefore a = 1, r = 1/2 \therefore \frac{1}{1-1/2} = \frac{1}{(1/2)} = \underline{2}$

3) Show that the infinite series $\frac{3}{10} + \frac{3}{100} + \frac{3}{100} + \dots$ is a geometric series by writing it in the form $\sum_{K=0}^{\infty} ar^K$ Calculate the sum

$= \frac{3}{10}\left(1 + \frac{1}{10} + \frac{1}{100} + \dots\right) = \sum_{K=0}^{\infty}(3/10)\left(\frac{1}{10^K}\right) \to \frac{a}{1-r} = \frac{3/10}{1-1/10} = \frac{3/10}{9/10} = \underline{1/3}$

4) Find the sum of the series $\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots = \sum_{K=0}^{\infty} \frac{1}{10}\left[\frac{1}{10}\right]^K$

$\to \frac{a}{1-r} = \frac{1/10}{9/10} = \underline{\frac{1}{9}}$

5) Express the infinite repeating decimal 0.444... as a single fraction by first writing it as a geometric series. $= \sum_{K=0}^{\infty} \frac{4}{10}\left(\frac{1}{10}\right)^K = \frac{4/10}{1-1/10} = \underline{\frac{4}{9}}$

6) Determine whether the geometric series is convergent or divergent. If, convergent, find the sum.

(a) $5 - 10/3 + 20/9 - 40/27 + \dots$

$\sum_{K=0}^{\infty} 5(-2/3)^K$

$-1 < r < 1$, so it converges

$\frac{a}{1-r} \to \frac{5}{1-(-2/3)}$ $\frac{5}{5/3} = 3$

(b) $1 + .4 + .16 + 0.064 + \dots$

$\sum_{K=0}^{\infty} 1\left(\frac{2}{5}\right)^K$ $a = 1, r < 1 = 2/5$

$\frac{1}{1-2/5} = \frac{1}{3/5} = \underline{\frac{5}{3}}$

c) $\sum_{K=0}^{\infty} \frac{1}{(\sqrt{2})^K}$ $-1 < 1/\sqrt{2} < 1$

$a = 1, r = 1/\sqrt{2}$

$\frac{1}{1-1/\sqrt{2}} = \frac{1}{\left(\frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)}$

$= \frac{1}{\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)} = \frac{\sqrt{2}}{\sqrt{2}-1}\left(\frac{\sqrt{2}+1}{\sqrt{2}+1}\right)$

$= \frac{2+\sqrt{2}}{(2-1)} = \underline{2+\sqrt{2}}$

(d) $\sum_{K=0}^{\infty} \frac{\pi^K}{3^{K+1}}$ $\frac{1}{3}\sum_{K=0}^{\infty} \frac{\pi}{3}^K$

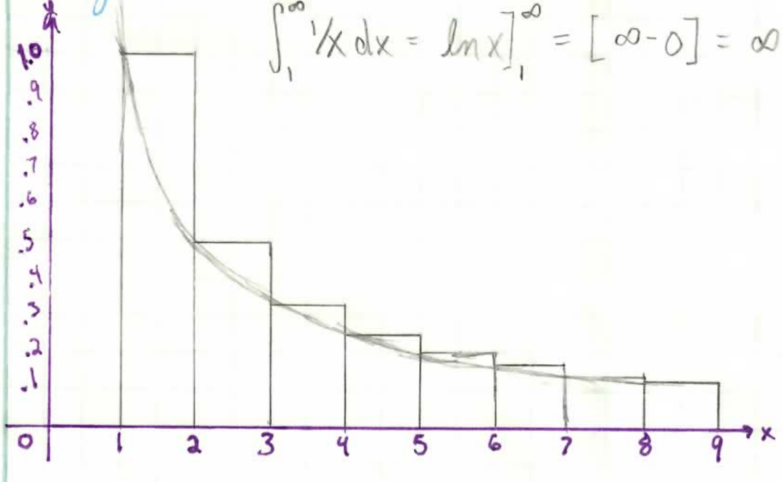
$r > 1$ no answer

I will be a math major.

Taylor Section 3 Skip Lester Feb 27 2013

7) The series $\sum_{k=1}^{\infty} \frac{1}{K} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ looks like it should converge, but in fact diverges. Prove that this series cannot have a finite sum. $\left\{ S_n > \int_1^{n+1} \frac{1}{x}\, dx \right.$

$\int_1^{\infty} 1/x \, dx = \ln x \Big]_1^{\infty} = [\infty - 0] = \infty$ and $S_n > \int dx/x$, $\therefore S_n = \infty$



Taylor Section 4 Skip Lester February 27, 2013

4/4

1) Consider the taylor series for $F(x)=\sin(-x)$.

(a) express the series in summation notation.

$$T(x)=\sum_{k=0}^{\infty}\frac{f^{k}(a)}{k!}(x-a)^{k} \qquad a=0$$

	$f(a)=$
$\sin(-x)$	0
$-\cos(-x)$	-1
$-\sin(-x)$	0
$\cos(-x)$	1
$+\sin(-x)$	0

$$=f(a)+f'(a)(x-a)+f''(a)(x-a)^2(1/2!)+f'''(a)(x-a)^3(1/3!)+\cdots$$

$$=0+(-1)(x)+0+(1)(x^3)(1/3!)$$

$$=0-x+0+x^3/6+0-x^5/5!$$

$$=-x+\frac{x^3}{3!}-\frac{x^5}{5!}=\sum_{k=0}^{\infty}(-1)^{k}\frac{x^{(2k+1)}}{(2k+1)!}$$

(b) Find an expression for $|R_3(x)|$. $R_3(x)\leq\frac{M_4}{4!}|x|^4=1/24|x|^4$.

c) Keeping in mind the proof that $\lim_{n\to\infty}\frac{x^n}{n!}=0$, show that $|R_n(x)|\to 0$ as $n\to\infty$.

$$\lim_{n\to\infty}|R_n(x)|\leq\lim_{n\to\infty}\frac{M_{n+1}}{(n+1)!}|x-a|^{n+1}$$

2) Find the taylor series for $F(x)=\cos(\frac{x}{2})$ and show that it converges to $\cos(\frac{x}{2})$ for all values of x. $T_n(x)=\sum_{k=0}^{\infty}\frac{F^{k}(a)}{k!}|x-a|^{k}$ $a=0$

$k=0$	$\cos(\frac{x}{2})$	1
1	$-\sin(\frac{x}{2})(1/2)$	0
2	$-\cos(x/2)(1/2)^2$	$-1/4$
3	$\sin(x/2)(1/2)^3$	0
4	$\cos(x/2)(1/2)^4$	$1/16$

$$=\sum_{k=0}^{\infty}(-1)^{k}\left(\frac{(x/2)^{(2k)}}{(2k)!}\right)$$

$$=\sum_{k=0}^{\infty}(-1)^{k}\left(\frac{x^{2k}}{2^{2k}(2k!)}\right)$$

$$1\cdot\lim_{n\to\infty}\frac{x^{n+1}}{(n+1)!}=1\cdot 0=0$$

$$|F(x)-T_n(x)|=|R_n(x)|\leq\frac{M_{n+1}}{(n+1)!}|x-a|^{n+1}=\frac{1\cdot x^{n+1}}{(n+1)!}\to 0$$

3) Find the taylor series for $f(x)=e^{x+2}$ and show that it converges to e^{x+2} for all x.

$$\frac{d}{dx}e^{x+2}=e^{x+2}$$

$k=0$	$F(0)=e^2$
$k=1$	$F'(0)=e^2$
$k=2$	$F''(0)=e^2$

$$\sum_{k=0}^{\infty}\frac{F^{k}(0)}{k!}|x-0|^{k}\Rightarrow\underline{\sum_{k=0}^{\infty}\frac{e^2x^k}{k!}}$$

$$e^2+e^2(x)+e^2(x^2)(1/2!)+e^2(x^3)(1/3!)$$

$$M_{n+1}=e^{x+2}$$

$$\hookrightarrow\lim_{n\to\infty}|R_n(x)|\leq\lim_{n\to\infty}\frac{M_{n+1}}{(n+1)!}(x)^{n+1}$$

$$\leq\lim_{n\to\infty}\frac{e^{x+2}|x|^{n+1}}{(n+1)!}$$

$$=(e^{x+2}\cdot 0)=0$$

4) For what values of x does the taylor series for f(x) converge to $f(x) = \frac{1}{1-2x}$? $|2x| < 1$ $|x| < 1/2$, or $x \in (-1/2, 1/2)$

$f(x) = (1-2x)^{-1}$ $f(0) = 1$

$f'(x) = -(1-2x)^{-2} \cdot -2$

$= \frac{2}{(1-2x)^2}$ $f'(0) = 2$

$f''(x) = -2 \cdot 2(1-2x)^{-3} \cdot -2$

$= \frac{8}{(1-2x)^3}$ $f''(0) = 8$

$f'''(x) = -3 \cdot 8(1-2x)^{-4} \cdot -2 \Rightarrow f'''(0) = 48$

$= \frac{48}{(1-2x)^4}$

$\Rightarrow T_n(x) =$

$1(x)^0 + 2(x) + 8(x^2)/2! + 48x^3/3! + \ldots$

$= x + 2x + 4x^2 + 8x^3 + \ldots$

$= \sum_{k=0}^{\infty} (2x)^k$

$\boxed{R = 1/2}$

Find the taylor and its R of C

Simple substitution... Taylor Section 5 Skip Lester February 28, 2013

1.) $\cos(2x) \Rightarrow \cos u$

$= 1 - \frac{u^2}{2!} + \frac{u^4}{4!} - \frac{u^6}{6!} + \dots = \sum_{k=0}^{\infty} (-1)^k \frac{u^{2k}}{(2k)!}$

$\rightarrow 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \dots = \sum_{k=0}^{\infty} (-1) \frac{(2x)^{2k}}{(2k)!} = \underline{\sum_{k=0}^{\infty} (-1) \frac{2^{2k} x^{2k}}{(2k)!} \quad R = \infty \text{ still}}$

2) $\frac{1}{1+x^2} \rightarrow \frac{1}{1-u}$

$= 1 + u + u^2 + u^3 + \dots = \sum_{k=0}^{\infty} u^k \quad |u| < 1$

$\rightarrow 1 + (-x^2) + (-x^2)^2 + \underbrace{(-x^2)^3}_{} \text{ sign flipping } \quad |x^2| < 1$

$= 1 - x^2 + x^4 - x^6 = \underline{\sum_{k=0}^{\infty} (-1)^k [x^{2k}], \; |x| < 1}$

$\underline{R = 1}$

(3) $e^{x^2} \rightarrow e^u$

$= 1 + u + \frac{u^2}{2!} + \frac{u^3}{3!} + \dots = \sum_{k=0}^{\infty} \frac{u^k}{k!}$

$\rightarrow 1 + (x^2) + \frac{(x^2)^2}{2!} + \frac{(x^2)^3}{3!} + \frac{(x^2)^4}{4!} + \dots$

$= 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} \dots = \underline{\sum_{k=0}^{\infty} \frac{x^{2k}}{k!}}$

Radius of convergence = ∞

4) $\frac{1}{1-6x} \rightarrow \frac{1}{1-u}$ let $u = 6x$

$= \sum_{k=0}^{\infty} u^k, \; |u| < 1, \; R = 1$

$= \underline{\sum_{k=0}^{\infty} (6x)^k \quad |x| < 1/6, \; R = 1/6}$

//multiplying both sides by the same object//

(5) $\frac{x^3}{1-x} = x^3 \cdot \frac{1}{1-x}$

$= x^3 \cdot [1 + x + x^2 + x^3 + x^4 + \dots] \quad = \sum_{k=0}^{\infty} x^{3+k}, \; |x^3| < 1 \quad R = 1$

$= x^3 + x^4 + x^5 + x^6 + x^7 + x \dots \qquad = |x| < 1$

6) $\frac{8x}{x+1} = 8x \cdot \frac{1}{1+x}$

$= 8x[1 - x + x^2 - x^3 + x^4 - \dots]$

$= 8x - 8x^2 + 8x^3 - 8x^4 + 8x^5 - \dots$

$= \underline{8 \sum_{k=0}^{\infty} (-1)^k (x)^{(k+1)}, \; |x| < 0, \; R = 1}$

(7) $x^5 \sin x$

$= x^5 \left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right]$

$= x^6 - \frac{x^8}{3!} + \frac{x^{10}}{5!} - \dots$

$= \underline{\sum_{k=0}^{\infty} (-1)^k \frac{x^{(2k+6)}}{(2k+1)!}}$

$R = \infty$

(8) $\frac{2}{x-1} = -2 \cdot \frac{1}{1-x}$

$= -2 \sum_{k=0}^{\infty} x^k \quad R = 0?$

Differentiating both sides

9) $\frac{1}{1-x^2} = \frac{d}{dx} \frac{-1}{1-x} = -1 \cdot \frac{1}{1-x}$

$= -1(1 + x + x^2 + x^3 + x^4 + \dots)$

$\frac{d}{dx}[-1 - x - x^2 - x^3 - x^4 - \dots]$

$\Rightarrow -1 - 2x - 3x^2 - 4x^3 - \dots$

$= -\sum_{k=0}^{\infty} (k+1)(x)^k$

(10) find the derivative of sin(x) with taylor

$\sin(x) = \sum_{k=0}^{\infty} (-1)^k \left(\frac{u^{2k+1}}{(2k+1)!} \right)$

$\frac{d}{dx} \left[x - \frac{x^3}{3!} + \frac{u^5}{5!} - \frac{u^7}{7!} + \dots \right]$

$\Rightarrow \underline{1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{u^6}{6!} + \dots} = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!} = \cos x$

Section 5 Calculus II: Taylor Series Skip Lester February 28, 2013

11) Find the derivative of cos x, by differentiating the taylor series for cos x.

$$\cos x \to \sum_{K=0}^{\infty} (-1)^K \left(\frac{x^{2K}}{(2K)!}\right) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\frac{d}{dx} \Rightarrow 0 - x + \frac{x^3}{3!} - \frac{x^5}{5!} = -\sum_{K=0}^{\infty} (-1)^K \left(\frac{x^{(2K+1)}}{(2K+1)!}\right) = -\sin x$$

12) Find the derivative of e^x, by differentiating the taylor series for e^x

$$e^x \to \sum_{K=0}^{\infty} \frac{x^K}{K!} = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right) \xRightarrow{d/dx} 0 + 1 + x + x^2/2! = e^x$$

//Integrating both sides//

13) Using taylor series, find the anti-derivative, [redacted] F(x), of sin(x), F(0)=0

$$\sin(x) \to \left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right] \quad F(x) = \left[\int x\,dx - \int \frac{x^3}{3!}\,dx + \int \frac{x^5}{5!}\,dx\right] \quad -\cos x = 0 \Leftrightarrow x = 1, -1$$

$$-\cos 0 = -1$$

$$= x^2/2! - x^4/4! + x^6/6! + C \to -1 + x^2/2! - x^4/4! + \cdots$$

$$= -\sum_{K=0}^{\infty} (-1)^K \frac{x^{2K}}{(2K)!} = -\cos(x)$$

14) " anti derivative, F(x), of cos(x)

$F(0) = 1$

$$\cos(x) \to 1 - \frac{u^2}{2!} + \frac{u^4}{4!} - \frac{u^6}{6!}$$

$$F(x) = \int \cos(x)\,dx = \int 1\,dx - \int \frac{x^2}{2!}\,dx + \int \frac{x^4}{4!}\,dx$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} \Rightarrow \sin(x)$$

15) Using Taylor series, find the antiderivative, for e^x

$$= 1 + x + x^2/2 + x^3/3! + \cdots$$

$$F(x) = \int 1\,dx + \int x\,dx + \int x^2/2\,dx + \int \frac{x^3}{3!}\,dx$$

$$= x + x^2/2! + x^3/3! + x^4/4! + C = 0$$

$C = 1$

$$= 1 + x + x^2/2! + \cdots = \sum_{K=0}^{\infty} \frac{u^K}{K!}$$

16) $\frac{1}{1+4x^2} \to \frac{1}{1-u} \to 1 + u + u^2 + u^3$

let $(-4x^2) = u$

$$1 + (-4x^2) + (-4x^2)^2 + (-4x^2)^3$$

$$= 1 - 4x^2 + 4^2x^4 - 4^3x^6$$

$$= \sum_{K=0}^{\infty} (-1)^K (4x^2)^{2K} = \sum_{K=0}^{\infty} (-4)^K (x)^{2K}$$

17) $\ln(1+x)$

$$= \int \frac{1}{(1+x)}$$

$$= \int (1 - x + x^2 - x^3 + \cdots)$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + C$$

$$= \sum_{K=0}^{\infty} (-1)^K \frac{x^K}{K}$$

18) $\frac{1}{4+x^2} \longrightarrow \frac{1}{1+u} =$

$$1 - u + u^2 - u^3 + u^4 - \cdots$$

$$\to \frac{1}{4}\left(\frac{1}{1+(x^2/4)}\right)$$

$$= 1 - (x^2/4) + (x^2/4)^2 - (x^2/4)^3 + \cdots$$

$$= \frac{1}{4} \sum_{K=0}^{\infty} (-1)^K \frac{x^{2K}}{4^{K+1}}$$

19) $\arctan(x) \to \frac{1}{1+u}$ w/ $\arctan(x) = \int \frac{1}{1+x^2}$

$$\to = \int (1 - u + u^2 - u^3)\,dx \to u - u^2/2 + u^3/3 - u^4/4$$

$$= x^2 - x^4/2 + x^6/3 - \cdots = \sum_{K=1}^{\infty} (-1)^K \left(\frac{x^{2K}}{K}\right)$$

Chapter 7.1 2,3,5,7,9 5 March 2013 Skip Lester Calculus II

2) Verify that $y = -t\cos t - t$ is a solution of the initial-value problem

$$t\frac{dy}{dt} = y + t^2\sin t \qquad y(\pi) = 0$$

$F(x)g'(x) + g(x)F'(x)$ let $F(t) = -t$

$g(t) = \cos t$

$$\frac{dy}{dt} = \left[-t \cdot -\sin t + \cos t \cdot -1\right] - 1$$

$$= t\sin t - \cos t - 1$$

$$t(t\sin t - \cos t - 1) = -t\cos t - t + t^2\sin t$$

$$= t^2\sin t - t\cos t - t \ \square$$

---

3) (a) For what values of r does the function $y = e^{rx}$ satisfy the differential equation $2y'' + y' - y = 0$ $\quad y = e^{rx} \rightarrow y' = re^{rx},\ y'' = r^2e^{rx}$

(b) If r_1 and r_2 are the values of r that you found in part (a), show that every member of the family of functions $y = ae^{r_1x} + be^{r_2x}$ is also a solution.

(a) $= 2(r^2e^{rx}) + re^{rx} - e^{rx} = 0$

$$e^{rx}(2r^2 + r - 1) = 0$$

$e^{rx} \neq 0 \quad 2r^2 + r - 1 = 0$

$ax^2 + bx + C \quad a = 2, b = 1, C = -1$

$$r = \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)} = \frac{-1 \pm \sqrt{9}}{4} = \frac{-1-3}{4} = -1$$

$$\frac{-1+3}{4} = \frac{1}{2}$$

$r = -1,\ r = 1/2$

(b) let $r_1 = 1/2$ & $r_2 = -1$

$$y = ae^{(x/2)} + be^{-x} \qquad 2y'' + y' - y = 0$$

$$y' = a/2\,e^{x/2} - be^{-x}$$

$$y'' = a/4\,e^{x/2} + be^{-x}$$

$$2\left(\frac{a}{4}e^{(x/2)} + be^{-x}\right) + \left(\frac{a}{2}e^{x/2} - be^{-x}\right) - \left(ae^{x/2} + be^{-x}\right) = 0$$

$$= \frac{a}{2}e^{x/2} + 2be^{-x} + \frac{a}{2}e^{x/2} - be^{-x} - ae^{x/2} - be^{-x} = 0$$

$$= \left[e^{x/2}\left(\frac{a}{2} + \frac{a}{2} - a\right) + e^{-x}(2b - b - b)\right] = 0$$

$$= 0e^{x/2} + 0e^{-x} = 0 \ \square$$

---

5) Which of the following functions are solutions of the differential equation $y'' + y = \sin x$?

$y'' + y = \sin x$

$\cancel{(a)}$ $y = \sin x$

$y'' = -\sin x$

$y'' + y = 0$

$\cancel{(b)}$ $y = \cos x$

$y'' = -\cos x$

$y'' + y = 0$

(c) $y = \frac{1}{2}x\sin x$

$$y' = \frac{x}{2}\cos x + \frac{1}{2}\sin x$$

$$y'' = \frac{1}{2}\cos x - \frac{x}{2}\sin x + \frac{1}{2}\cos x$$

$$= \cos x - \frac{x}{2}\sin x$$

$$y'' + y = \cos x$$

(d) $-\frac{1}{2}x\cos x$

$$y' = -\frac{1}{2}\cos x + \frac{x}{2}\sin x$$

$$y'' = \frac{1}{2}\sin x + \frac{1}{2}\sin x + \frac{x}{2}\cos x$$

$$= \sin x + \frac{x}{2}\cos x$$

$y'' + y = \sin x$ ✓

7, 9, 10-14 Calculus II Chapter 7.1 Skip Lester March 5, 2013

7) (a) what can you say about a solution of the equation $y'=-y^2$ just by looking at the equation? Its derivative is negative, so y is decreasing on all ℝ except perhaps 0.

(b) verify that all members of the family $y=\frac{1}{x+c}$ are solutions for (a)

$y=(x+C)^{-1}$ $y'=-(x+c)^{-2}=\frac{-1}{(x+C)^2}$; $\left(\frac{1}{x+C}\right)^2=\left((x+c)^{-1}\right)^2 \therefore \frac{d}{dx}(x+c)^{-1}=-\left[(x+c)^{-1}\right]^2$

(C) Can you think of a solution of the differential equation $y'=-y^2$ that is not a member of the family in (b)? $y=0$

(d) find a solution of the initial-value problem $y'=-y^2$// ~~$y(\pi)=0$~~ $y(0)=0.5$

$y=\frac{1}{x+2}$ because $f(0)=1/0+2$ meets our initial condition.

9) a population is modeled by the differential equation

$$\frac{dP}{dt}=1.2P\left(1-\frac{P}{4200}\right)$$

(a) For what values of, P is the population increasing? $P\in(0,4200)$

(b) for what values decreasing? $P\in(4200,\infty)$ ☹

(C) What are the equilibrium solutions? $P=0$, $P=4200$

10) a function $y(t)$ satisfies the differential equation

$$\frac{dy}{dt}=y^4-6y^3+5y^2$$

(a) what are the constant solutions? 0, 1, 5

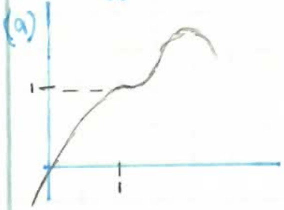
$0=y^2(y^2-6y+5)$

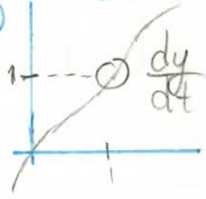
(b) for what values of y is y increasing? (c) decreasing?

$0<y^2-6y+5$

$0=(y-5)(y-1)$

11) Explain why the functions with the given graphs (p.499) can't be solutions of the differential equation $\frac{dy}{dt} = e^t(y-1)^2$ $= 0 \Leftrightarrow y=1$ $e^t(y-1)^2$ is always increasing.



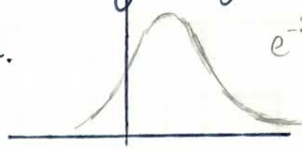


$\emptyset$ $\frac{dy}{dt} \neq 0 \Leftrightarrow y=1$

the graph of (a) is decreasing in places, so it will not work.

12) The function with the given graph is a solution of one of the following differential equations. Decide which & explain.

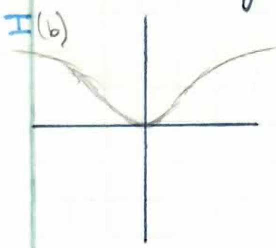
A. $y'=1+xy$ B. $y'=-2xy$ [✓ C. $y'=1-2xy$]

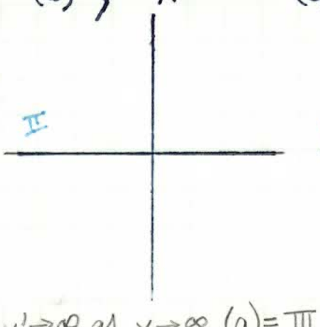


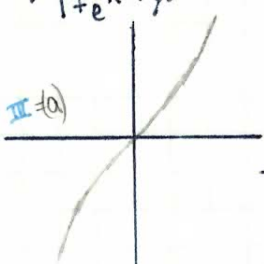
Can't be B because when x=0, our derivative is not 0. By that line of reasoning, our derivative is 0 to the right of origin and above y=0 in Q I. 1+xy could never be zero in Quadrant I, so A. doesn't work. By process of elimination, C.

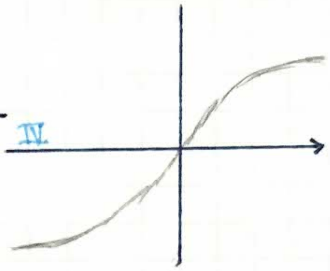
13) Match the differential equations with the solution graphs labeled I-IV. Explain.

(a) $y'=1+x^2+y^2$ (b) $y'=xe^{(-x^2-y^2)}$ (c) $y'=\frac{1}{1+e^{x^2+y^2}}$ (d) $y'=\sin(xy)\cos(xy)$









(a) $y' \geq 1$ on $\mathbb{R}$ and $y' \to \infty$ as $x \to \infty$. (a)=III

(b) $y'<0 \Leftrightarrow x<0$ + $y'>0 \Leftrightarrow x>0$, and $y'=0 \Leftrightarrow x=0$. I=(b)

(c) $(1+e^{x^2+y^2})^{-1} = y' \to 0$ as $x \to \infty$, (c)=IV

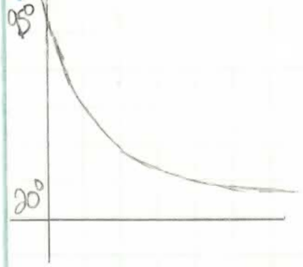
(d) = II

14) Suppose you have just poured a cup of freshly brewed coffee with temperature 95°C in a room where the temperature is 20°C.

(a) When do you think the coffee cools most quickly? What happens to the rate of cooling as time elapses? At time = 0; rate of cooling is decreasing as time increases.

(b) Newton's Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its ambiance. let a = ambient Temp.

Write as a D.E. $T' = \frac{dT}{dt} = \underset{\text{decreasing}}{-}K\underbrace{(T-a)}_{\text{difference between object temp + ambient temp}}, \quad T(0) = T\text{-}0$ let $T = \text{Temp}_{\text{object}}$



1, 3-7, 9-19 odd | Chapter 7.2 | Skip Lester 3/07/2013

1) a direction field for the differential equation $y' = x\cos \pi y$ is shown.

a) Sketch graphs of the solutions that satisfy the initial conditions.

i) $y(0)=0$, ii) $y(0)=0.5$ iii) $y(0)=1$ iv) $y(0)=1.6$

b) Find all equilibrium solutions. see paper

From the graph, it appears that $y = 1/2$ and $y = 3/2$ are equilibrium solutions

$x^3 \sin y = \frac{dy}{dx}$

$x^2 \sin y\, dx = dy$

$x^2 dx = \frac{dy}{\sin y}$

3-6 Match the DE to its D.F

3) $y' = 2 - y$ III, equilibrium at $y = 2$ + slope independent of x-vals

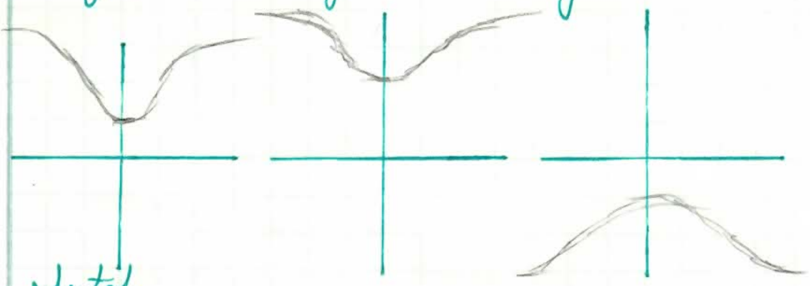
4) $y' = x(2-y)$ I, equilibrium at $y = 2$, $dy/dx = 0$

5) $y' = x + y - 1$ IV, dy/dt is largest in Q I and largest neg at Q III

6) $y' = \sin x \sin y$ II, dy/dx is $0 \Leftrightarrow x$ or $y = \pi$

7) Use the direction field labeled II to sketch the graphs of the solutions that satisfy the given initial conditions.

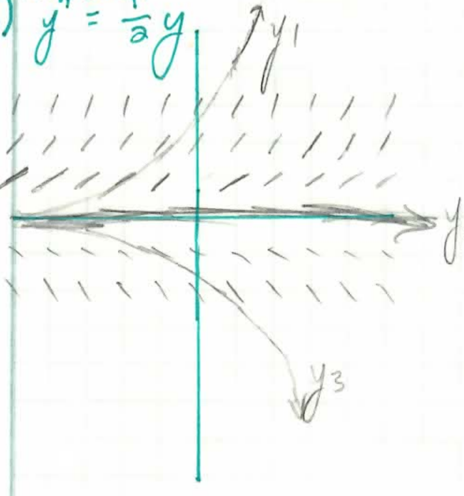
(a) $y(0)=1$ (b) $y(0)=2$ (c) $y(0)=-1$



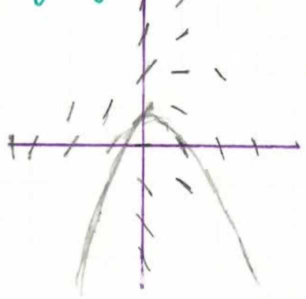
⑬ $y' = y + xy$ $(0,1)$

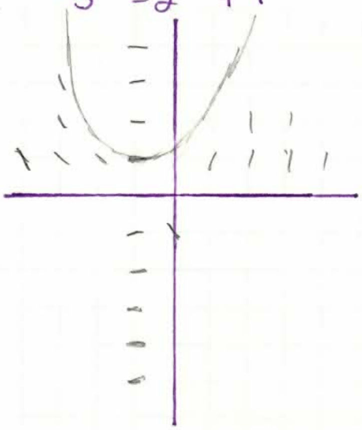
x	y	y'
0	±2	±2
1	±2	±4
-3	±2	∓4

9) sketch $y' = \frac{1}{2}y$



11) $y' = y - 2x$ $(1,0)$





Chapter 7.2

Skip Lester Calculus II

March 06, 2013

19) Use Eulers method with each of the following step sizes to estimate the value of $y(0.4)$, where y is the solution of the initial-value proflem $y' = y$, $y(0) = 1$.

(i) $y' = F(x,y) = y$ and $y(0) = 1$

$x_0 = 0$, $y_0 = 1$

(i) $h = 0.4$ (ii) $h = 0.2$ (iii) $h = 0.1$

(i) $hF(x_0, y_0) + y_0 = y_1$ (ii) $h = 0.2 \rightarrow x_1 = 0.2$ and $x_2 = 0.4$

$.4(1) + 1 = 1.4$

20) $y_n = y_{n-1} + hF(x_{n-1}, y_{n-1})$, $y(x_0) = y_0$ [print out]

$x_0 = 0$ $x_1 = 1$?

$y_{(1)} = 0 + hF(0,0)$

21) Use Euler's method with step size 0.5 to compute the approximate y-values $y_1, y_2, y_3,$ and y_4 of the solution of the initial value proflem

$y' = y - 2x$, $y(1) = 0$

$h = 0.5$ $F(x,y) = y - 2x$

$x_0 = 1$, $y_0 = 1$

$x_1 = x_0 + h = 1 + 0.5 = 1.5$

$x_2 = 2$

$x_3 = 2.5$

$y_1 = y_0 + hF(x_0, y_0)$

$= 0 + \frac{1}{2}F(1,0)$ $0 - 2(1)$

$\underline{y_1 = \frac{1}{2}[-2] = -1}$

$y_2 = y_1 + hF(x_1, y_1)$

$y_2 = -1 + \frac{1}{2}F(\frac{3}{2}, -1)$ $(-1 - 2(3/2))$

$= -1 + \frac{1}{2}(-4)$ -4

$\underline{y_2 = -3}$

$y_3 = y_2 + hF(x_2, y_2)$

$y_3 = -3 + \frac{1}{2}F(2, -3)$

$-3 + -7/2$ $[-3 - 2(2)]$

-7

$\underline{y_3 = -13/2}$

$y_4 = y_3 + \frac{1}{2}F(x_3, y_3)$

$= -13/2 + 1/2(5/2, -13/2)$

$= -13/2 + \frac{1}{2}(-\frac{23}{2})$ $[-13/2 - 2(5/2)]$

$\frac{-13-10}{2} = -\frac{23}{2}$

$= -\frac{13}{2} - \frac{23}{4}$

$y_4 = -26 - 23 = \underline{\frac{-49}{4}} = y_4$

23, 25, 28 | 7.2 | Skip Lester March 8 2013

23) Use Eulers method with step size = 0.1 to estimate $y(0.5)$, where $y(x)$ is the solution of the initial value problem

$y' = y + xy \quad , y(0) = 1$

$x_0 = 0,$ $\quad y_1 = y_0 + hF(x_0, y_0)$

$x_1 = 0.1$ $\quad y_1 = 1 + .1[1 + 0.1]$
$= 1.1$

$x_2 = 0.2$ $\quad y_2 = 1.1 + 0.1[1.1 + 0.1 \cdot 1.1]$
$= 1.221$

$x_3 = 0.3$

$x_4 = 0.4$ $\quad y_3 = 1.221 + \frac{1}{10}[1.221 + (0.2 \cdot 1.221)]$
$= 1.36752$

$x_5 = 0.5$

$y_4 = y_3 + hF(x_3, y_3)$
$= 1.36752 + .1(1.36752 + (0.3 \cdot 1.36752))$

$y_4 = 1.5452976$

$y_5 = y_4 + hF(x_4, y_4)$

$y_5 = 1.5452976 + \frac{1}{10}(1.5452976 + (.4 \cdot 1.5452976))$

$\underline{y_5 = 1.76139264}$

25) Calculator

28) From ex 7.1 #14, our coffee now cools at a rate of 1°C per minute when its temperature is 70°C. $\frac{dT}{dt} = -K(T - R)$

(a) what does the differential equation become in this case?

$\frac{dT}{dt} = -1°C \rightarrow \quad \frac{dT}{dt} = -K(70 - 20) = -1 \therefore K = -1/50 \rightarrow \frac{dT}{dt} = \frac{-1}{50}(T - 20)$

(b) Sketch a solution curve to the initial value problem (plot the D. Field) What is the limiting value of the temperature?

~~Utilisez~~
(c) ~~Allez~~-vous les méthode ~~de~~ d'Euler avec $h = 2$ pour estimer la température
Utilisez

du café après 10 minutes. 81.15

Solve the differential equation Chapter 7.3 Separable Equations Skip Lester March 10, 2013

5/5

1.)

$\frac{dy}{dx} = xy^2$

$\Rightarrow \frac{dy}{y^2} = x\,dx \quad (y \neq 0)$

$\int \frac{1}{y^2}\,dy = \int x\,dx$

$\frac{-1}{y} + C = \frac{x^2}{2} + C$

$y^{-1} = -\frac{x^2}{2} - C$

$y = \frac{1}{-\frac{1}{2}x^2 - C}$

$\underline{y = \frac{2}{K - x^2}, \quad K = -2C}$

trivial solution $y=0$

9) $\frac{du}{dt} = 2 + 2u + t + tu$

$\frac{du}{dt} = (2+t)(1+u)$

$\frac{du}{(1+u)} = (2+t)\,dt$

$\int \frac{du}{1+u} = \int (2+t)\,dt$

$\ln|1+u| = 2t + t^2/2 + C$

$|1+u| = e^{(2t + t^2/2 + C)}$

$1+u = \pm e^{(t^2/2 + 2t + C)}$

$\underline{u = -1 \pm e^{(t^2/2 + 2t + C)}}$

3) $(x^2+1)\,y' = xy$

$(x^2+1)\frac{dy}{dx} = xy$

$\frac{dy}{y} = \frac{x}{(x^2+1)}\,dx$

$\Rightarrow \int \frac{dy}{y} = \int \frac{x}{(x^2+1)}\,dx \quad$ let $u = x^2+1$, $du = 2x\,dx$

$\Rightarrow \ln|y| = \frac{1}{2}\ln(x^2+1) + C$

$= \ln|y| = \ln(x^2+1)^{(1/2)} + \ln e^C$

$= \ln|y| = \ln(e^C\sqrt{x^2+1})$

$|y| = e^C\sqrt{x^2+1}$

$\underline{y = K\sqrt{x^2+1}, \quad K = \pm e^C}$

(11) Find the solution that satisfies the given initial condition.

$\frac{dy}{dt} = \frac{t}{y} \quad y(0) = -3$

$y\,dy = t\,dt \rightarrow \int y\,dy = \int t\,dt$

$\frac{y^2}{2} = \frac{t^2}{2} + C$

$\frac{-3^2}{2} = \frac{0^2}{2} + C$

$C = \frac{9}{2}$

$\frac{1}{2}y^2 = \frac{1}{2}t^2 + \frac{9}{2}$

$y = -\sqrt{t^2+9}$

$y(0) = -\sqrt{9} = -3$ ✓

5) $(y + \sin y)\,y' = x + x^3$

$(y + \sin y)\frac{dy}{dx} = x + x^3$

$\int y\,dy + \int \sin y\,dy = \int x + x^3\,dx$

$\underline{\frac{y^2}{2} + \cos y = \frac{x^2}{2} + \frac{x^4}{4} + C} \ \square$

(7) $\frac{dy}{dt} = \frac{te^t}{y\sqrt{1+y^2}}$

$y\sqrt{1+y^2}\,dy = te^t\,dt$

$\int y(1+y^2)^{(1/2)}\,dy = \int te^t\,dt$

let $u = (1+y^2)$, $du = 2y\,dy$

$= \frac{1}{2}\int u^{(1/2)}\,du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{1}{3}(1+y^2)^{3/2} = te^t - e^t + C$

$1 + y^2 = (3te^t - 3e^t + 3C)^{2/3} - 1$

$y = \sqrt{(3te^t - 3e^t + 3C)^{2/3} - 1}$

$\underline{y = \sqrt{3(te^t - e^t + C)^{2/3} - 1}}$

13) $\frac{du}{dt} = \frac{2t + \sec^2 t}{2u} \quad u(0) = -5$

$\int 2u\,du = \int (2t + \sec^2 t)\,dt$

$u^2 = t^2 + \tan t + C$

$u = \pm\sqrt{t^2 + \tan t + C}$

$-5 = \pm\sqrt{0^2 + \tan 0 + C} \quad C = 25$

$\underline{u = -\sqrt{t^2 + \tan t + 25}}$

Chapter 7.3 Skip Lester March 10 2013

21) Solve the differential equation $y'=x+y$ by making the change of variable

$u=x+y \rightarrow \frac{d}{dx}(u)=\frac{d}{dx}(x+y) \rightarrow \frac{du}{dx}=1+\frac{dy}{dx}$; $\frac{dy}{dx}=x+y=u$

$\rightarrow \frac{du}{dx}=1+u \rightarrow \frac{du}{(1+u)}=dx \rightarrow \int\frac{du}{(1+u)}=\int dx \rightarrow \ln|1+u|=x+c$

oops... 23 next Page

$|1+u|=e^{x+c} \rightarrow 1+u=\pm e^{x+c} \rightarrow u=\pm e^{c}e^{x}-1 \rightarrow x+y=\pm e^{c}e^{x}-1 \rightarrow \underline{y=\pm e^{c}e^{x}-x-1}$

25) C.A.S. Solve the initial-value problem $y'=\frac{\sin x}{\sin y}$, $y(0)=\pi/2$

$\frac{dy}{dx}=\frac{\sin x}{\sin y} \rightarrow \int \sin y\,dy=\int\sin x\,dx \rightarrow -\cos y=-\cos x+C$ but $\cos(\pi/2)=\cos(0)-C$

$0=1-C$; $C=1$

$\cos y=\cos x-1 \rightarrow y=\cos^{-1}(\cos x-1)$

29) Find the orthogonal trajectories of the family of curves. $4yy'=-2x$

$x^2+2y^2=k^2 \Rightarrow \frac{d}{dx}(x^2+2y^2)=\frac{d}{dx}k^2 \Rightarrow 2x+4yy'=0$

$y'=\frac{-2x}{4y}=\frac{-x}{2y}$

Orthogonal = reciprocal of tangent;

Orthogonal trajectories must follow $y'=\frac{2y}{x} \Rightarrow \frac{dy}{dx}=\frac{2y}{x}$

$\int\frac{dy}{y}=\int\frac{2dx}{x} \rightarrow \ln|y|=2\ln|x|+c \rightarrow \ln|y|=\ln|x|^2+c \rightarrow |y|=e^{\ln|x|^2+c}$

$y=\pm x^2\cdot e^{c}=Cx^2$ (parabola)

32) $y=\frac{x}{1+kx}$ $\frac{(1+kx)\cdot 1-x(k)}{(1+kx)^2}=y'$ $\text{orth}\rightarrow -\frac{(1+kx^2)}{(1+kx)-xk}=y' \rightarrow$ $y=\int\frac{-(1+kx^2)}{(1+kx)-xk}\,dx$

$y_{orth}=-x-\frac{kx^3}{3}$ (via mathematica)

Chapter 7.3 | Skip Lester March 11, 2013

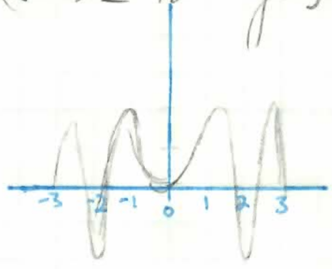
#23)

(a) Solve the differential equation $y'=2x\sqrt{1-y^2}$

$\frac{dy}{d\cancel{x}} = 2x\sqrt{1-y^2} \longrightarrow \frac{dy}{\sqrt{1-y^2}} = 2x\,dx \rightarrow \int \frac{1}{1-y^2}\,dy = \int 2x\,dx$

(b) $\sin^{-1}y = x^2+C$, $-\pi/2 \leq (x^2+C) \leq \pi/2$ $\quad y(0)=0 \quad \sin^{-1}0 = 0^2+C \rightarrow C=0$

$\sin^{-1}y = x^2 \rightarrow y = \sin(x^2)$



(c) $y(0)=2$ DNE

$-1 \leq y \leq 1$ for all x

35) An integral equation is an equation that contains an unknown function $y(x)$ and an integral that involves $y(x)$. Solve. Use an initial condition obtained from the integral equation.

$y(x) = 4 + \int_0^x 2t\sqrt{y(t)}\,dt \rightarrow y(0) = 4 + \int_0^0 2t\sqrt{y(t)}\,dt = 4+0=4$

$\frac{dy}{d\cancel{x}} = 2x\sqrt{y} \longleftrightarrow y'(x) = 2x\sqrt{y(x)}$

$\frac{dy}{\sqrt{y}} = 2x\,dx \rightarrow \int \frac{dy}{\sqrt{y}} = \int 2x\,dx$

$\int y^{-(1/2)}\,dy = x^2+C$

$2\sqrt{y} = x^2+C \quad C=4[(y(0)=4)]$

$2\sqrt{4} = 0+C \quad C=4$

$\sqrt{y} = \frac{x^2+4}{2}$

$y = \left(\frac{x^2}{2}+2\right)^2$

Chapter 7.3

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#37) $\frac{dQ}{dt} = (12-4Q)$ Amps

$\int \frac{dQ}{12-4Q} = \int dt$

$= \ln|12-4Q| \cdot \frac{-1}{4} = t+c$

$\frac{-\ln|12-4Q|}{4} = t+C$

$\ln|12-4Q| = -4t-4c$

$|12-4Q| = e^{(-4t-4c)}$

$4Q = 12 \pm e^{(-4t-4c)}$

$Q = \frac{-12 \pm e^{(-4t-4c)}}{4}$

$Q = \frac{12 - Ke^{-4t}}{4}$ $K = \pm e^{-4c}$

$Q(t) = 3 - Ae^{-4t}$ $A = K/4$

$Q(0) = 0$ $Q = 3 - A$

$A = 3$

$Q(t) = 3 - 3e^{-4t}$.

$\lim_{t \to \infty} Q(t) = 3 - 0 = 3C$

39)

$\frac{dP}{dt} = K(M-P)$ *find an expression for P(t)*

$K > 0$ *find the limit*

~~$\frac{dP}{K(M-P)} = dt \rightarrow \frac{1}{K}\int \frac{1}{(M-P)} dP = \int dt$~~

~~$\frac{1}{K} \cdot \ln|M-P| = t \rightarrow \ln|M-P| = Kt$~~

~~$|M-P| = e^{Kt}$~~ $\rightarrow P = M \pm e^{Kt}$

$\int \frac{dP}{(P-M)} = \int -K\,dt \rightarrow \ln|P-M| = -Kt + C$

$|P-M| = e^{-Kt+C} = e^C e^{-Kt} = Ae^{-Kt}$ $A = \pm e^C$

$P = M + Ae^{-Kt}$ Beginners suck, so $P(0) = 0$

$P = M + Ae^{0} = 0$ $A = -M$

$P = M - Me^{-Kt}$ $\lim_{t \to \infty} (M - Me^{-Kt}) = M$

Practice makes perfect

1-5, 9, 11, 13-15, 19, 20 | Chapter 7.4 Exponential Growth + Decay | Skip Lester | March 12, 2013

1) a population of protozoa develops with a constant relative growth rate of 0.7944 per members per day. On day zero, the population consists of two members. Find the population size after six days.

$K = \frac{1}{P}\frac{dP}{dt} = 0.7944$, $\frac{dP}{dt} = 0.7944P$. $P(t) = P(0) = P_0 e^{0.7944t} = 2e^{0.7944t}$

$P(6) = 2e^{(0.7944(6))} \approx 235$ members

2) a common inhabitant of human intestines is the bacterium E. coli. a cell of this bacterium in a nutrient-broth medium divides into two cells every twenty minutes. The initial population of a culture is 60 cells.

a) Find the relative growth rate. $P(t) = P_0 e^{kT} \Rightarrow P(0) = 60e^{k(0)}$

$P(1/3) = 120 = 60e^{k(1/3)}$

$\quad /60 \quad /60$

$2 = e^{k(1/3)}$

$\ln 2 = k(1/3)$

$\frac{1}{P}\frac{dP}{dt} = \underline{3\ln 2 = k}$

b) Find an expression for the number of cells after t hours. $P(t) = 60e^{(3\ln 2 \cdot t)}$

c) Find the number of cells after 8 hours

$P(8) = 60e^{(3\ln 2 \cdot 8)} = 1.00663 \times 10^9$ cells.

d) Find the rate of growth after 8 hours. $P'(t) = 124.76e^{3\ln 2 \cdot t}$

$\frac{dP}{dT} = 3\ln 2 \cdot P(8) = f'(8) \approx 2.09323 \times 10^9$

e) when will the population reach 20,000 cells.

$\frac{20{,}000}{60} = \frac{60e^{(3\ln 2 t)}}{60}$

$\ln\left(\frac{20{,}000}{60}\right) = (3\ln 2)T$

$\frac{\ln\left(\frac{2000}{6}\right)}{3\ln 2} = t \approx 2.79$ hours

3-5, 9, 11, 13-15, 19, 20 Chapter 7.4 Skip Lester March 13, 2013

3) a bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 420. $P(t) = P_0 e^{Kt} \Rightarrow P(0) = 100e^{K(0)}$ $\frac{P(1) = 100e^{K\cdot 1}}{100} = \frac{420}{100}$

a) find an expression for the # of bacteria after t hours $\Rightarrow P(t) = 100e^{(\ln 4.2t)} = \underline{100(4.2)^t}$

$e^K = 4.2$
$K = \ln 4.2$

b) find the # of bacteria after 3 hours $P(3) = 100(4.2)^3 = 7408.8 \approx \underline{7409 \text{ bacteria}}$

c) find the rate of growth after 3 hours. $\frac{dP}{dt} = KP \rightarrow P'(3) = K\cdot P(3)$
$= \ln 4.2 \cdot (100(4.2)^3)$
$\approx \underline{10,632 \text{ bac/hour}}$

d) $\frac{P(t) = 100(4.2)^t}{100} = \frac{10,000}{100} \rightarrow 4.2^t = 100$

$\ln(4.2)\cdot t = \ln 100$

$t = \frac{\ln(100)}{\ln(4.2)} \approx \underline{3.2 \text{ hours}}$

4) a bacteria grows with constant relative growth rate. The bacteria count was 400 after 2 hours and 25,600 after 6 hours.

a) What is the relative growth rate? $P(t) = P_0 e^{Kt}$ $P(2) = 400 = P_0 e^{K2}$

$P(6) = 25600 = P_0 e^{K6}$ $P(0) = 400e^{Kt}$ $P(4) = 25600 = 400e^{K4}$

$\frac{\ln 64}{4} = K = 1.03972$

$64 = e^{K4}$
$\frac{\ln 64}{4} = \frac{K4}{4}$

b) what is the initial size of the culture?

$P(-2) = 400e^{K(-2)} = \underline{50 \text{ bacteria}} \rightarrow 50e^{(\ln 64/2)} = 400\checkmark$

$P(0) = 50e^{(\ln 64/4)(0)} \rightarrow P(2) = 50e^{(\ln 64/2)} = 400$

c) $P(t) = 50e^{(\ln 64/4)(t)}$ (d) $P(4.5) = 50e^{(\ln 64/4)(4.5)} \approx 5382$ cells

e) $\frac{dP}{dt} = KP \rightarrow K\cdot P(4.5) = 5595.5$ bacteria / hour

f) $\frac{50{,}000}{50} = \frac{50e^{(\ln 64/4)(t)}}{50}$

$1000 = e^{(\ln 64/4)(t)}$ $t = \frac{\ln 1000}{\left(\frac{\ln(64)}{4}\right)} = 6.64$ hours

$\ln 1000 = (\ln 64/4)(t)$

5, 9, 11, 13, 14, 15, 19, 20

Chapter 7.4

Skip Lester March 13, 2013

5)

year	Population
1750	790
1800	980
1850	1260
1900	1650
1950	2560
2000	6080

The table gives estimates of the world population, in millions. (a) Use the exponential model and the figures for 1750 & 1800 to predict the world population in 1900 and 1950. Compare to actual figures

Let the population (in millions) at year t be $P(t)$

$P(t) = P(1750)e^{K(t-1750)}$; $P(1800) = 980 = 790e^{K(1800-1750)}$ $\frac{980}{790} = \frac{790e^{K(1800-1750)}}{790}$

$\ln\left(\frac{980}{790}\right) = \ln(e^{50K}) = 50k$

1900 - 1750 = 150
1950 - 1750 = 200

$K = \ln\left(\frac{980}{790}\right) \cdot \frac{1}{50} \approx 0.0043104$

$$\left.\begin{aligned} P(1900) &= 790e^{K(150)} = \underline{1506 \text{ million}} \\ P(1950) &= 790e^{K200} \simeq \underline{1871 \text{ million}} \end{aligned}\right\} \text{Underestimates}$$

(b) Use the exponential model and population figures for 1850 + 1900 to predict 1950.

$P(t) = P(1850)e^{K(t-1850)}$

$P(1900) = 1260e^{K(50)}$

$\ln\left(\frac{1650}{1260}\right) = \ln e^{K50} = K50$

$K = \frac{1}{50} \cdot \ln\left(\frac{1650}{1260}\right) \approx 0.005393$

$P(1950) = 1260e^{(0.005393 \cdot 100)}$

$= \underline{2161 \text{ million}}$

still under-estimating

(C) use the figures for 1900 and 1950 to predict population in 2000

$P(t) = P(1900)e^{K(t-1900}$

$= 1650e^{K(t-1900)}$

$P(1950) = 1650e^{K(50)}$

$\frac{2560}{1650} = e^{K50}$

$K = \frac{1}{50}\ln\left(\frac{2560}{1650}\right) \approx 0.008785$

$P(2000) = 1650e^{K(100)}$

$\approx \underline{3972}$ mil very low under-estimate

(9) The half-life of cesium 137 is 30 years. Suppose we have a 100mg sample.

Let $y(t)$ = mass in mg remaining after t years. $y(t) = y_0e^{kt} = 100e^{kt}$

$y(30) = 100e^{30K} = 50$

(b) How much remains after 100 years? $100 \cdot 2^{(-100/3)} \approx \underline{9.92 \text{ mg}}$

$e^{30K} = 1/2$

(C) after how long will only 1 mg remain?

$K = (1/30)\ln(1/2)$

$K = -\ln 2/30$

$1 = 100e^{(-\ln 2t/30)} \rightarrow \ln\left(\frac{1}{100}\right) = \frac{-\ln 2t}{30}$

$t = \frac{-30}{\ln 2}\left(\ln\frac{1}{100}\right) = \underline{199.3 \text{ years}}$

(a)

$y(t) = 100e^{(-\ln 2\, t/30)} = \underline{100 \cdot 2^{-t/30}}$

Chapter 7.4 Skip Lester March 13, 2013

11) Scientists can determine the age of ancient objects with carbon-dating. The half-life of ^{14}C is 5730 years. The level of ~~radioactivity~~ radioactivity decreases exponentially. A parchment fragment was discovered with 74% as much ^{14}C radioactivity ~~of~~ as plants today. How old is this parchment?

let $y(t)$ be the level of radioactivity. $y(t)=y(0)e^{-Kt}$. $y(5730)=\frac{1}{2}y(0)$

$$y(0)e^{-K(5730)}=\frac{1}{2}y(0)$$

$$e^{-K(5730)}=1/2$$

$$-5730K=\ln 1/2=-\ln 2$$

$$K=\frac{\ln 2}{5730}$$

$$y(t)=.74\,y(0)$$

$$0.74=e^{-t(\ln 2/5730)}$$

$$\ln(0.74)=-t\left(\frac{\ln 2}{5730}\right)$$

$$t=\frac{-5730}{\ln 2}(\ln 0.74)\approx \underline{2489\text{ years}}$$

13) a roast turkey is taken from an oven when its temperature has reached 185°F and is placed on a table in a room where the temperature is 75°F

(a) if the turkey temp. is 150°F after 30 minutes, what is it after 45 minutes?

(b) when will it have cooled to 100°? $\frac{dT}{dt}=K(T-T_s)=K(T-75)$

$$y=T-75$$

$$y(0)=T(0)-75$$

$$y(0)=185-75$$

$$y(0)=110$$

$$\frac{dy}{dt}=Ky \quad y(t)=y(0)e^{Kt}$$

$$y(t)=110e^{Kt}$$

$$y(30)=150-75=75=110e^{K30}$$

$$\ln\left(\frac{75}{110}\right)=30K$$

$$K=\frac{1}{30}\left(\ln\left(\frac{75}{110}\right)\right)\approx-0.0127664$$

$$y(t)=110e^{(-0.0127664t)}$$

$$y(45)=110e^{K(45)}\approx 62°F$$

$$62+75=\underline{T(45)=137°F}$$

(b)

$$100=T(t)\rightarrow y(t)=25$$

$$25=110e^{(1/30\ln 75/110)t}$$

$$\ln\frac{25}{110}=\frac{1}{30}\ln\left(\frac{15}{22}\right)$$

$$t=\frac{30\ln(25/110)}{\ln(15/22)}\approx 116.06\text{ minutes}$$

Skip Lester

14) a corpse was 32.5°C at 1:30 P.m. and 30.3°C an hour later. Normal body temperature is 37.0°C and the surrounding air temp was 20°C. When did dude die? $t=0, T=37$

$$\frac{dT}{dt} = K(T-T_s) \to K(T-20)$$

$$\frac{dT}{(T-20)} = K dt$$ separable

$$\ln|T-20| = Kt + C$$

$$t-20 = e^{Kt+C} = e^c e^{Kt} = Ae^{Kt}$$

$$t-20 = Ae^{Kt}$$

$$T = 20 + Ae^{KT}$$

$$T(0) = 20 + Ae^{(0)}$$

$$37 = 20 + A$$

$$A = 17$$

$$T = 20 + 17e^{KT}$$

$$32.5 = 20 + 17e^{Kt} \quad \frac{12.5}{17} = e^{Kt} \quad K = \frac{\ln\left(\frac{12.5}{17}\right)}{t}$$

$$30.3 = 20 + 17e^{K(t+1)}$$

$$30.3 = 20 + 17e^{\left(\ln\frac{12.5}{17}\cdot\frac{1}{t}\right)(t+1)}$$

$$\frac{10.3}{17} = e^{\left(\ln 12.5/17 \cdot \frac{1}{t}\right)(t+1)}$$

$$\ln\left(\frac{10.3}{17}\right) = \ln\left(e^{\ln(12.5/17)}\right)^{\left(\frac{t+1}{t}\right)}$$

$$\ln\left(\frac{10.3}{17}\right) = \left(\frac{t+1}{t}\right)\ln\left(\frac{12.5}{17}\right)$$

$$t\ln\left(\frac{10.3}{17}\right) = (t+1)\ln\left(\frac{12.5}{17}\right)$$

$$t\ln(10.3/17) = t\ln(12.5/17) + \ln(12.5/17)$$

$$-t\ln(12.5/17)$$

$$t(\ln(10.3/17) - \ln(12.5/17)) = \ln(12.5/17)$$

$$t = \frac{\ln(12.5/17)}{\ln\left(\frac{10.3}{17}\right) - \ln\left(\frac{12.5}{17}\right)} \approx 1.6 \text{ hours}$$

15, 19, 20 | Chapter 7.4 | Skip Lester March 12, 2013

15) When a cold drink is taken from a 'fridge, its temperature is 5°C. After 25 minutes in a 20°C room its temperature has increased to 10°C. (a) What is the temperature of the drink after 50 minutes? (b) When will its temperature be 15°C?

(a) by newtons law of cooling, $\frac{dT}{dt} = K(T - T_S) \rightarrow K(T-20)$, $y(t) = y_0 e^{Kt}$

$y(0) = T(0) - 20 \rightarrow 5 - 20 = -15$, $y(25) = -15e^{K25}$

$y(25) = T(25) - 20$

$y(25) = 10 - 20 = -10$

$-10 = -15e^{K25}$

$\ln\left(\frac{2}{3}\right) = K \cdot 25$ $K = \frac{1}{25}\ln(2/3)$

$y(t) = -15e^{\left(\frac{1}{25}\ln(2/3) \cdot t\right)} = 15 \cdot \frac{2}{3}^{t/25}$

$y(50) = -15 \cdot \left(\frac{2}{3}\right)^{(50/25)} = -15 \cdot \left(\frac{2}{3}\right)^2$

$= -\frac{60}{9} = -\frac{20}{3}$

$T(50) = 20 + y(50)$

$= 20 - \frac{20}{3}$

$= 40/3 = 13.\overline{3}°C$

(b)

$15 = T(t) = (20 + y(t))$

$15 = 20 - 15 \cdot \frac{2}{3}^{t/25}$

$\frac{15 \cdot \frac{2}{3}^{t/25}}{15} = \frac{5}{15}$

$\ln\left(\frac{2}{3}^{t/25}\right) = \ln\left(\frac{1}{3}\right)$

$t = \frac{25\ln\left(\frac{1}{3}\right)}{\ln(2/3)} = 67.74$ min

19)

The Logistic equation   Chapter 7.5 1,3,5,6,7,9,13,15 Skip Lester March 13, 2013

1) Suppose that a population develops according to the logistic equation $\frac{dP}{dt} = 0.05P - 0.0005P^2$, where t is measured in weeks.

(a) what is the carrying capacity? What is the value of K?

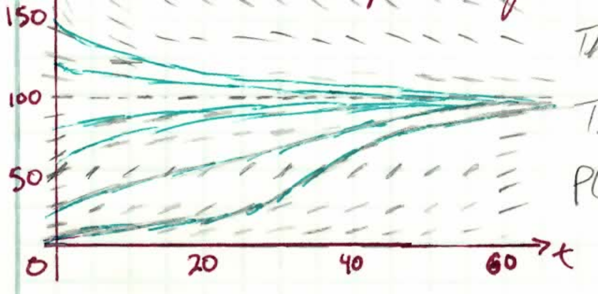
$$\frac{dP}{dt} = 0.05P - 0.0005P^2 \qquad \frac{dP}{dt} = KP\left(1 - \frac{P}{m}\right)$$

carrying capacity

$$= 0.05P(1 - 0.01P) \qquad \therefore K = \frac{1}{20},\ M = \frac{1}{100}$$

$$= \frac{1}{20}P\left(1 - \frac{P}{100}\right)$$

(b) For the direction field on page 538, Where are the slopes close to zero? Where are the slopes largest? Which solutions are increasing? decreasing?



The slopes approach 0 when P is 0 or 100

The largest slope is $\Leftrightarrow$ P = 50.

P(t) is increasing for 0<P<100

decreasing for P>100

C) Sketch solutions for initial populations of 20, 40, 60, 80, & 120, & 140. What do the solutions have in common? How do they differ? Which solutions have inflection points? At what populations do they occur?

They all approach P=100

Sign of slope.

P(20) has I.P. at P=50

(d) What are the equlibrium solutions? P=0, P=100

They have constant zero slope.

---

3) The pacific halibut fishery has been modelled by the differential equation

$\frac{dy}{dt} = Ky(1 - y/m)$, where y(t) is the biomass (in kg) per time t (years)

$M = 8\times10^{-7}$ kg, $K = 0.71$

(a) if $y(0) = 2\times10^7$ kg, find biomass a year later

(b) How long will it take the biomass to reach 4×10^7 Kg?

$$y(t) = \frac{M}{1+Ae^{-kt}},\ A = \frac{M - y(0)}{y(0)} \rightarrow \frac{8\times10^7}{1+3e^{-0.71(1)}}$$

$$= \frac{8-2}{2} \qquad \overset{(a)}{\approx} \underline{3.23\times10^7 \text{kg}}$$

$$A = 3$$

$$4\times10^7 = \frac{8\times10^7}{1+3e^{-0.71(t)}}$$

$$1+3e^{-0.71t} = 2$$

$$-1 \quad /3 \quad -1/3$$

$$\ln(e^{-0.71t}) = \ln\left(\frac{1}{3}\right)$$

$$t = \frac{-\ln(1/3)}{0.71} \approx \underline{1.55 \text{ years}}$$

The Logistic equation

Chapter 7.5 5, 6, 7, 9, 13, 15 Skip Lester March 13, 2013

5) Suppose a population grows according to a logistic model with initial population 1,000 and carrying capacity 10,000. If the population grows to 2,500 after one year, what will the population be after another 3 years? $A = \frac{M-P_0}{P_0} = \frac{10{,}000-1000}{1000} = 9, \therefore$ by (4) $P(t) = \frac{10{,}000}{1+9e^{-kt}}$

$P(1) = 2500$

$$2500 = \frac{10{,}000}{1+9e^{-K(1)}}$$

$$1+9e^{-K} = 4$$

$$-1 \qquad -1$$

$$9e^{-K} = 3$$

$$/9 \qquad /9$$

$$e^{-K} = 1/3$$

$$K = -\ln(1/3) = \ln(3)$$

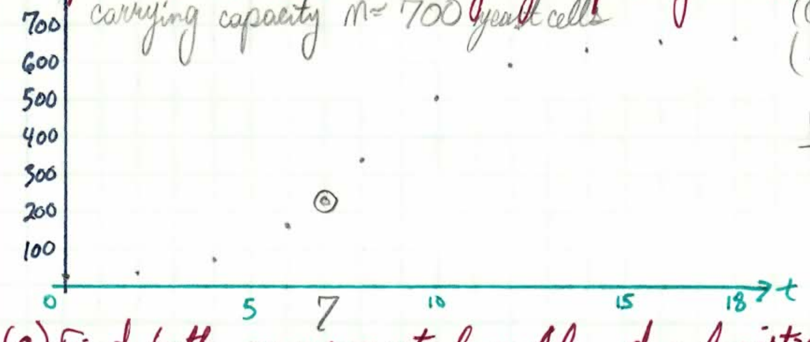
$$P(4) = \frac{10000}{1+9e^{-4\ln 3}} = \boxed{9{,}000}$$

(6) The table gives the number of yeast cells somewhere.

time	Yeast cells	time	Yeast cells
0	18	10	509
2	39	12	597
4	80	14	640
6	171	16	664
8	336	18	672

6(a) Plot the data and use the plot to estimate the carrying capacity. (b) and the initial growth rate.

carrying capacity M ≈ 700 yeast cells



(0, 18)
(2, 39)

$P' = kP$ $\quad \frac{39-18}{2} = 10.5$

$$\frac{P'}{P} = k = \frac{10.5}{18} = .58$$

$$K = .58\overline{3}$$

(c) Find both an exponential model and a logistic model for these data.

$$\frac{dP}{dt} = .58P \qquad P = e^{.58t+c}$$

$$P = 18e^{.58t}$$

$$\int \frac{dP}{P} = \int .58\,dt$$

$$\ln|P| = .58t + c$$

$$\frac{dP}{dt} = KP\left(1-\frac{P}{m}\right) \qquad A = \frac{700-18}{18}$$

$$= .58\overline{3}\,P\left(1-\frac{P}{700}\right) \qquad = 37.\overline{8}$$

$$P = \frac{700}{1+37.\overline{8}e^{(-.58t)}}$$

(e) Use your logistic model to estimate the number of yeast cells after 7 hours

$$P(7) = \frac{700}{1+37.\overline{8}e^{-(.58*7)}} = 427.47 \approx 427 \text{ yeast cells}$$

7.5 7,9,13,15 Skip Lester March 14, 2013

7) The population of the world was about 5.3 billion in 1990. Birth rates in the '90's ranged from 35 to 40 million per year and death rates ranged from 15 to 20 million per year. Lets assume the carrying capacity for the world population is 100 billion. Let $t=0=1990$

(a) Write a logistic differential equation for these data. Units in billions 1×10^9

Birth rate - Death rate = growth rate $\quad \frac{dP}{dt}=kP\left(1-\frac{P}{m}\right)$

$37-17=$ 20 million / year $=\frac{dP}{dt}=.2$

$K=\frac{1}{P}\frac{dP}{dt}=\frac{1}{5.3}(0.2)=\frac{1}{265}$ $\quad \underline{\frac{dP}{dt}=\frac{P}{265}\left(1-\frac{P}{100}\right)}$

(b) Use the logistic model to estimate the world pop. in 2000. Actual $=6.1\times10^9$. Compare

$P(t)=\frac{M}{1+Ae^{-Kt}}$ $\quad A=\frac{M-P_0}{P_0}=\frac{100-5.3}{5.3}=17.8679$

$=\frac{947}{53}$

$P(10)=\frac{100}{1+(17.87)e^{(-10/265)}}\approx$ $\underline{5.49 \text{ billion}}<6.1$ billion we're low

(c) Predict 2100 + 2500 $P(110)=7.81$ billion ; $P(510)=22.41$ billion

(d) What are your predictions w/ $M=50\rightarrow\frac{50-5.3}{5.3}=\frac{447}{53}=A$

$\frac{50}{1+(447/53)e^{(-t/265)}}$ $\quad P(10)=5.48$ billion $\quad P(510)=22.41$ lower!

$P(110)=7.61$ billion

---

9) One model for the spread of a rumor is that the rate of spread is proportional to the product of the fraction y of the pop. that has heard the rumor and the fraction that hasn't heard.

(a) $\frac{dy}{dt}=ky(1-y)$ (b) $\frac{dP}{dt}=kP\left(1+\frac{P}{m}\right)\rightarrow M\frac{dy}{dt}=k(my)(1-y)$

$y=P/m \quad P=my$ $\quad P(t)=\frac{M}{1+Ae^{-kt}}$ $\quad A=\frac{M-P_0}{P_0}$

(c) 7h 36m

$\frac{dP}{dt}=\frac{dy}{dt}\cdot m$

$My=\frac{M}{1+\frac{M-P_0}{P_0}e^{-kt}}\rightarrow y=\frac{y_0}{y_0+(1-y_0)e^{-kt}}$

7.5 13, 15 Skip Lester March 15, 2013

13) The table gives the midyear population of Japan, from 1960 to 2005

Population In THOUSANDS

year	1960	1965	1970	1975	1980	1985	1990	1995	2000	2005
population	94,092	98,883	104,345	111,573	116,807	120,754	123,537	125,341	126,700	127,417

We let $t=0=1960$ $\underline{P_E(T)}=1578.3(1.0933)^t+94000$

$\underline{P_L(T)} = \frac{32,658.5}{1+12.75e^{-0.1706(t)}}+94000$

P_E gets awfully exponential

P_L is probably more accurate

15) Lets modify the logistic differential equation of example 1 as follows:

$$\frac{dP}{dt} = 0.08P\left(1-\frac{P}{1000}\right)\boxed{-15}$$

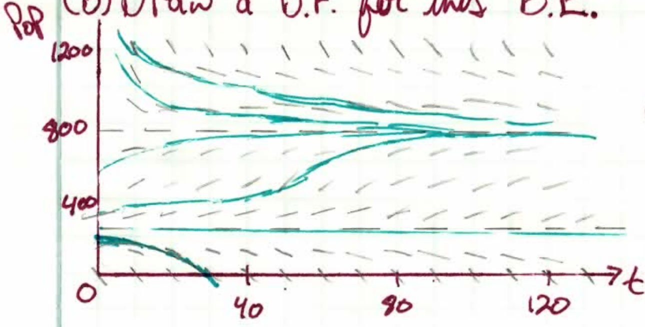
(a) Suppose P(t) represents fish population at time t, t in weeks.

Explain the meaning of the -15 term

(a) No matter what the population, 15 fish are "subtracted" from the pool

-15 /week

(b) Draw a D.F. for this D.E.



C) ~~equivalent equil~~

Equilibrium solutions $P\simeq 250, P\simeq 750$

d) An initial fish population of less than 250 fishies cannot survive.

Populations above 250 approach $P=750$

E) Using $P(0)=200$ and $P(0)=300$, Solve explicitly $250-750Ke^{t125}$

$$\frac{dP}{dt}=0.08P(1-P/1000)-15$$

$$-\frac{100,000}{8}\cdot\frac{dP}{dt} = \frac{-100,000}{8}\cdot\left[0.08P\left(1-\frac{P}{1000}\right)-15\right]$$

$$-12,500\frac{dP}{dt} = P^2-1000P+187,500$$

$$\frac{dP}{(P-250)(P-750)} = \frac{-1}{12,500}dt$$

1,3-7, 9   Chapter 7.6   Skip Lester March 17,2013

1. For each predator - prey system, determine which of the variables, x or y, represents the prey population and which represents the predators. Is the growth of the prey restricted just by the predators or by other factors as well? Do the predators feed only on prey or do they have additional food sources? Explain

(a) $\frac{dx}{dt} = -0.05x + 0.0001xy$

$\frac{dy}{dt} = 0.1y - 0.005xy$

$\frac{dx}{dt} \; -0.05x \longleftrightarrow y=0$

∴ a declining population x in the absence y indicates that x represents predators. Because y grows unchecked, we surmise that y is the prey.

(b) $\frac{dx}{dt} = 0.2x - 0.0002x^2 - .006XY$

$\frac{dy}{dt} = -0.015y + 0.00008xy$

In the absence of x, $\frac{dy}{dt}$ is declining, ∴ $\frac{dy}{dt}$ represents $\Delta P_{\text{predators}}$

$\frac{dx}{dt}$ must be prey

3. The system of differential equations

$\frac{dx}{dt} = 0.5x - 0.004x^2 - 0.001xy$

$= .5x(1 - x/125) - 0.001xy$

X has a carrying capacity $M = 125$

when x increases y decreases.

$\frac{dy}{dt} = 0.4y - 0.001y^2 - 0.002xy$

$.4y(1 - y/400) - 0.002xy$

y has carrying capacity $M=400$

When y increases, x decreases.

∴ X & y are in competition for resources.

4;5,6,7,9 Skip LesteR MARch 18,2013

4) Flies, frogs, and crocodiles exist in an environment. To survive, frogs need to eat flies and crocodiles need to eat frogs. In the absence of crocodiles and flies, the frog population will decay exponentially. If $P(t)$, $Q(t)$, and $R(t)$ represent the populations at time t, write a system of differential equations as a model for their evolution. If the constants in your equation, explain your choice of sign.

$\frac{dP}{dt} = CP - CPQ$ $\frac{dq}{dt} = -Cq + CqP - CqR$ $\frac{dR}{dt} = -Cr + Crq$

Skip Lester's cheat sheet

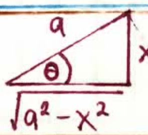
$|E_m| = \frac{K(b-a)^3}{24n^2}$ Midpoint $= \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$, $\Delta x = b-a/n$, $\bar{x}_i = (x_{i-1}+x_i)/2$

$|E_T| = \frac{K(b-a)^3}{12n^2}$ Trapezoid $= \frac{\Delta x}{2}\left[f(x_0) + 2f(x_i) + 2f(x_{i+1}) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$ $\Delta x = \frac{b-a}{n}$, $x_i = a + i\Delta x$

$|E_S| = \frac{K(b-a)^5}{180n^4}$ Simpson's $= \frac{\Delta x}{3}\left[f(x_0) + 4f(x_i) + 2f(x_{i+1}) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$

$\sin^2 x = \frac{1-\cos 2x}{2}$ ‖ $\cos^2 x = \frac{1+\cos 2x}{2}$ ‖ $\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\arctan\frac{x}{a}$ ‖ $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$

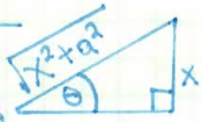
$\frac{d}{d\theta}\cos^{-1}(\theta) = \frac{-1}{\sqrt{1-\theta^2}}$ ‖ $\frac{d}{d\theta}\sec(\theta) = \sec\theta\tan\theta$ ‖ $\sqrt{a^2-x^2} \rightarrow a\sin\theta = x$



$\rightarrow \sqrt{a^2-x^2} = a\cos\theta$

$\sqrt{x^2+a^2}$

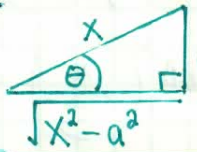
$x = a\tan\theta$



$\sqrt{x^2+a^2} = a\sec\theta$

$\sqrt{x^2-a^2}$

$a\sec\theta = x$

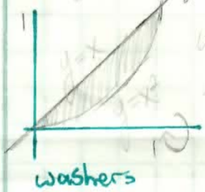


$\sqrt{x^2-a^2} = a\tan\theta$

Disks ⇒ $A = \pi(\text{radius})^2$ $\pi\int[f(x)]^2dx$

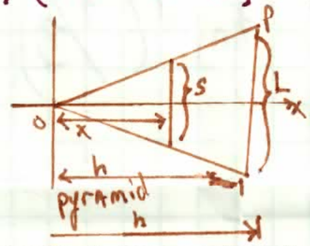
Washers ⇒ $A = \pi[(\text{outer radius})^2 - (\text{inner radius})^2]$

$\pi\int[f(x)]^2 - [g(x)]^2]dx$



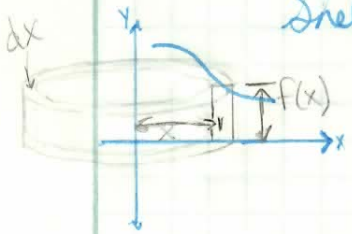
washers

$\pi\int_0^1(x-x^2)dx$



$V = \int_0^h A(x)dx = \int_0^h \frac{L^2}{h^2}x^2dx = \frac{L^2}{h^2}\frac{x^3}{3}\Big]_0^h = \frac{L^2h}{3}$

$\cos 2x = \cos^2 x - \sin^2 x$



Shells: $V = \int_a^b \underbrace{2\pi x}_{\text{circumference}} \overset{\text{height}}{f(x)} \overset{\text{Thickness}}{dx}$

Arc length:

for $x = f(t)$ and $y = g(t)$

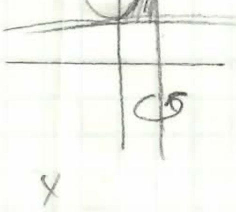
and $a \leq t \leq b$ is traversed once

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\,dy$$

$f_{ave} = \frac{1}{b-a}\int_a^b f(x)dx$



$\frac{1}{4}\int\frac{x}{(\cancel{x}+3)}$

$\int \frac{1}{x(x-1)^2}dx$

Skip's Cheat Sheet

Taylor Series

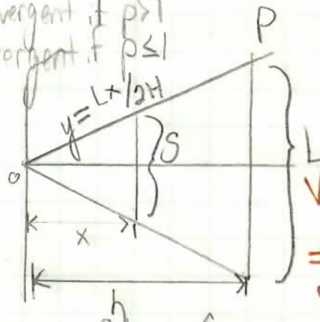
$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

degree n, centered at $x=a$

$$R_n(x) = f(x) - T_n(x) = \frac{M_{n+1}}{(n+1)!}|x-a|^{n+1}$$

convergent?

$\int_1^\infty \frac{1}{x^p}dx$ is convergent if $p>1$
divergent if $p\leq 1$



$$\frac{x}{h} = \frac{S/2}{L/2} = \frac{S}{L}$$

Pyramid w/ square base

$$V = \int_0^h A(x)\,dx = \int_0^h \frac{L^2}{h^2}x^2\,dx = \frac{L^2}{h^2}\frac{x^3}{3}\Big]_0^h = \frac{L^2h}{3}$$

$$V = \int_a^b A(x)\,dx$$

Disks: $A = \pi r^2$ Washers: $\pi[r_{out} - r_{in}]^2 = A$

Shells: $V = \int_a^b \underbrace{2\pi x}_{\text{circumference}} \underbrace{f(x)}_{\text{height}} \underbrace{dx}_{\text{thickness}}$ $0 \leq a < b$

Arc Length

watch your squares.

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$$L = \int_a^b \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\,dy$$

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Trig Stuff:

$\frac{d}{dx}$ $\cos^{-1}(x) \Rightarrow \frac{-1}{\sqrt{1-x^2}}$

$\sin^{-1}(x) \Rightarrow \frac{1}{\sqrt{1-x^2}}$

$\sec(\theta) \Rightarrow \sec\theta\tan\theta$

$\tan\theta \Rightarrow \sec^2\theta$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$P(t)\ \frac{K}{1+Ae^{-kt}}$$

$$\frac{dP}{dt} = kP\left(1-\frac{P}{K}\right)$$

$$A = \frac{K-P_0}{P_0}$$

Approximations

Midpoint $M_n(x) = \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$, $\Delta x = {}^{b-a}/_{n}$, $\bar{x}_i = \frac{(x_{i-1}+x_i)}{2}$

error $|E_m| = \frac{K(b-a)^3}{24n^2}$

Trapezoid $Trap_n(x) = \frac{\Delta x}{2}[f(x_0) + 2f(x_i) + \cdots + 2f(x_{n-1}) + f(x_n)]$

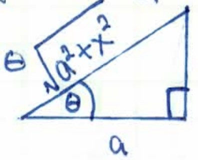
$\Delta x = \frac{b-a}{n}$ $x_i = a + i\Delta x$ $|E_T| = \frac{K(b-a)^3}{12n^2}$

Simpson's $S_n(x) = \frac{\Delta x}{3}[f(x_0) + 4(f(x_i)) + 2f(x_{i+1}) + \cdots$
$+ 2(f(x_{n-2})) + 4(f(x_{n-1})) + f(x_n)]$

$|E_S| = \frac{K(b-a)^5}{180n^4}$

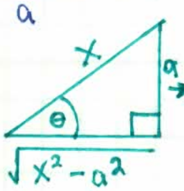
$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\arctan\frac{x}{a}$$

$x = a\tan\theta$



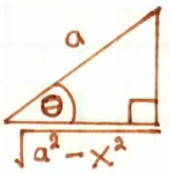
$x \rightarrow \sqrt{x^2+a^2} = a\sec\theta$

$x = a\sec\theta$



$a \rightarrow \sqrt{x^2-a^2} = a\tan\theta$

$x = a\sin\theta$



$x \rightarrow \sqrt{a^2-x^2} = a\cos\theta$

Euler's Method $y_n = y_{n-1} + hF(x_{n-1}, y_{n-1})$, $n = 1, 2, 3 \ldots$ h=step

$\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$ if even, 0 if odd.

$$g(t) = \int_0^x f(t)dt$$

$$g'(x) = f(x)$$